CPL WORKING PAPERS

This volume contains various papers concerning CPL. The principal one of these is an unfinished draft of the Reference Manual. This is preceded by a copy of an elementary programming manual and brief notes on the history, present status and future prospects of the language and its compilers, and is followed by various appendices.

These documents are not intended for publication in their present state. They are being given a limited private circulation in this incomplete form for reasons discussed in the preface.

July 1966.
Preface

In October 1962 a joint research project was begun by the University Mathematical Laboratory, Cambridge and the Institute of Computer Science, London (then known as the University of London Computer Unit). The aim of this project was to design and implement a new programming language for use on the Atlas I and II (Titan) computers in the two establishments.


It was intended that the language should possess the advantages of the general structure and precise description of Algol 60 but should be of wider practical utility. Many of its main features were settled at an early stage and were described (in 1963) in a paper in the Computer Journal (ref. 1).

Since the publication of the initial description the language has been subject to extensive redesign and further research work has been carried out on its semantics (see refs. 2, 3).

The design of the language, as opposed to its implementation, has never been more than a part time occupation for any of the authors. As the language gradually approached its present state, the authors' meetings became less frequent and those who were not actively engaged in an implementation found it more difficult to keep in touch with the language. At the same time some of the people who were engaged in an implementation began making substantial contributions to the language itself. During the period 1963 to 1966 D.W. Barron left the CPL language group and D. Park and M. Richards from Cambridge and G.F. Coulouris from London joined it.

The proper description of a programming language is no easy task, and CPL, which is very considerably larger and more sophisticated than Algol, presents a formidable problem. None of the authors have been able to spare the time to document the language adequately, and the fact that it has been evolving continuously has not simplified the problem.

An Elementary Programming manual exists in several versions, the latest of which is included in this volume, but this only describes the simpler parts of the language. Three chapters of an Advanced Programming manual were written (by C.S) but these, too, only cover the easier parts of the language and by now need considerable revision.

In Spring 1965, therefore, the authors decided to prepare and if possible publish a Reference Manual which should contain a complete description of CPL as it then stood. It was not intended that the Reference Manual should be an introductory text and no particular pains were to be taken to make it easy to read by the uninitiated; it was hoped, however, that the result would not be as difficult as the Algol 60 Report.
The first draft was prepared by JNB after a series of long meetings at which the authors discussed the points at issue. The second draft was then prepared by JNB, DFH, MR, GPC, and DP, each writing one or more sections. These were then revised, edited and partly rewritten by CS to produce the incomplete draft which forms the main body of this volume.

At a meeting in June 1966 the present authors (JNB, GPC, DFH, EN, DP, MR and CS) decided that the present volume should be made available to a restricted audience as a research report. The chief reason for this was that many of the authors by now had too many other commitments to undertake any further work on the language. At the same time it was generally appreciated that at least two important additions to the language (Compound Data Structures and Segmentation) were essential if CPL were to be of wide application. With the very limited effort available it was felt that an interim publication of an unfinished document which could be carried out with very little work was all that could be expected if the vital work of developing the language additions was not to be delayed too long to be incorporated in a new implementation which MR hopes to start at MIT in the autumn of 1966. The present volume is the result.

Major developments of CPL are to be expected in the future. In particular a scheme for incorporating Compound Data Structures in a very general way is under development. It is also intended to consider the problems of segmentation and, more generally, of the relationship of the language to its compiler and operating system. Other, less wholesale changes are under consideration and a list of these is given below as an Appendix.

If these developments prove successful we would hope to extend and revise the current draft of the Reference Manual and, in due course, to publish it.

References


CPL REFERENCE MANUAL

(Incomplete Draft)

July 1966
Editor's Apologia

The unfinished draft which follows shows imperfections of several kinds:

1. Missing sections

1.3.4, 1.3.5, 9.2.3, are missing merely because they have not yet been written. They presented some problems in exposition and so were postponed.

1.4, 9.5.1, 9.5.2 were postponed because the content was still under discussion.

Appendices 2 and 3 are still under discussion.

2. Incorrect Sections

The whole of section 2 is an unrevised earlier draft and may need major revision.

9.5 is now largely wrong as we have altered the rules about break, return and result is to allow a more general use. This is too complicated to insert as a manuscript alteration.

3.2, 3.4.6 and 6.5 will need modification when the type character is introduced.

4.6 and 8.3.5 will need modification when the scope of where-clauses is changed.

3. Errors, Misprints and Infelicities

Some of these have been corrected in manuscript. I should be very grateful for a note of any more which come to light.

4. Defects of Style

The draft inevitably still shows signs of its original multiple authorship in the varying styles of the different sections. More serious, perhaps, is the vacillation between a concise definition-like style (such as used in the Algol 60 report) and a more relaxed expository style. I find myself temperamentally averse from writing in English in a very formalised manner, so that the longer the draft has been under my care, the longer it has become. The only cure I can see for this is the development of a precise and symbolically expressed formal theory of semantics which still eludes our grasp.
5. Defects of Exposition

These are probably frequent and are only partly inadvertent. The Reference Manual is not intended to be an exposition or explication of CPL. The definitions of the meaning or effect of some operator or command in terms of other or simpler constructions may be in error in prescribing undesirable actions in infrequent cases. As these are discovered they will be altered.

It is part of our general approach to CPL that logical coherence and convenience in use are not to be sacrificed to brevity or simplicity in explanation.

In the ordinary course of events I should have been most unwilling to publish a report with so many defects. However, in the circumstances, given the desirability of circulating more information about CPL at once and the equally great urgency of working on Compound Data Structures together with the fact that none of the authors has time to spare on the Reference Manual, there seems to be nothing else to do.

I hope it will be possible to publish a revised and corrected version of the Reference Manual incorporating the treatment of Compound Data Structures on which we are now working in the not too distant future. It would be of great assistance to me if readers who discover errors of any sort in this draft would send me a list of them. If a sufficiently large number appear, we shall try to circulate a correction sheet to everyone who has a copy of this report.

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July 1966
INTRODUCTION

This report is intended to be a complete description of CPL as at present defined. The syntax of the language is given using a system based on that used for ALGOL 60 (see Section 1.2). Semantic descriptions are given in words and by example (see Section 1.3). As a satisfactory formal language for describing semantics has not yet emerged, this document should not be regarded as completely rigorous; it should be read with a modicum of common sense.

One of the principal aims in designing CPL was to make it a practical application of a logically coherent theory of programming languages. It is not the purpose of this report to expound the underlying theory. However, some parts of this report, and in particular, the semantic descriptions, have been much simplified by making use of some of its concepts. As these may be unfamiliar, a brief general discussion of some of the issues involved, particularly as they apply to CPL, has been included in Section 1. Section 2 contains an informal account of the relationship between Publication CPL and the more formal Canonical CPL which underlies the publication language.

Part II (Sections 3-11) contain the description of Canonical CPL which is the main purpose of this report.
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NOTE Sections 1.3.4, 1.3.5 and the whole of 1.4 are not yet written.
1.1.1. Commands and Expressions

An ALGORITHM is a rule for computation, using the words in a wide and informal sense. A PROGRAM is an algorithm presented in a form in which it can be accepted and, hopefully, executed by a computing machine. A PROGRAMMING LANGUAGE is the formalism in which a program is expressed.

Algorithms are built up in an hierarchical manner (which is not usually described formally) from components which are imperative sentences. These sentences contain verbs and nouns or descriptive phrases which take the place of nouns. For example in the imperative sentence:

"Replace x by the product of x and y."

the phrase "the product of x and y" is a description which is the second object of the verb "replace".

One of the ways in which the hierarchy is built up is by using one algorithm as part of a descriptive phrase used in another. The example just given might have been written "Replace x by the result of multiplying x by y." Here "multiply x by y" is a subsidiary imperative which has become incorporated in the descriptive phrase "the result of....". In this case, however, there is some ambiguity as it is not entirely clear if the command "multiply x by y" means "replace x by the result of multiplying x by y" or merely "discover the result of multiplying x by y". Such ambiguities cannot be tolerated in a program, and it is part of the function of a programming language to make their elimination simple and safe.

The features discussed above make their appearance in all programming languages, though in some the ambiguity has not been wholly eliminated. The terms used in CPL to describe them are the following.

A COMMAND corresponds to an imperative sentence.

An EXPRESSION corresponds to a descriptive phrase.

A NAME corresponds to a noun and is a special case of an expression.

The mechanism for incorporating the result of one algorithm in a descriptive phrase of another is completely explicit (see Section 4.4).
1.1.2 Equivalence of Algorithms

There is no generally accepted rule for determining the equivalence of two algorithms independently of their context. There is, however, one situation when it is quite clear what should be meant by such an equivalence. If an algorithm, \( A \), occurs as part of the descriptive phrase "... the result of ..." as a component of a larger algorithm, then we are only interested in the "result" of \( A \) and any other algorithm, \( B \), which produces exactly the same result will be acceptable in its place. In these circumstances we can say that \( A \) and \( B \) are locally equivalent.

1.1.3 Equivalence of Programs

Two programs are said to be equivalent if they produce indistinguishable effects whenever they are executed. This execution must be complete, not partial, so that, for instance the intermediate effects of two equivalent programs may differ, and in particular all programs which do not terminate may be regarded as equivalent. The question whether two given programs are equivalent or not in this sense is \( \text{\textit{\textbf{undecidable}}\text{\textbf{}}} \), but in certain simple cases it may be possible to prove equivalence by one of the following rules.

1. Certain commands are defined in this manual to be equivalent to other, generally simpler, commands or sequences of these.

2. Two commands which differ only in their component parts (which may be expressions or other commands) are equivalent if each of the corresponding component parts of the two commands are equivalent.

These are not the only possible cases in which commands are equivalent. The whole question of equivalence of programs is a complicated and not yet fully understood one and its investigation lies outside the scope of this report.

(Note that equality of routines as defined in section 10.2.4 is a completely different and much stronger concept than that of equivalence between programs.)
1.1.4 Equivalence of Expressions

Two expressions are equivalent when they have the same value. In the case of a block-expression the value of the whole expression is the value of the expression following the basic symbol result is so that any "side effects" such as assignments to non-local variables are ignored when determining equivalence.

When determining the equivalence of expressions involving standard mathematical operators only the properties of the ideal mathematical operators are relevant. Departures from the ideal owing to the fact that only finite representations are used in a computer, are to be ignored. Thus, for example, the expressions \( x + y - z \) and \( y - z + x \) are to be considered as equivalent irrespective of the precision or range of the internal representations of numbers, and irrespective of the order of evaluation of the components.

1.1.5 Rearrangement Rules

The general rule in CPL is permissive. Any program or expression may be replaced by any other which is equivalent to it in the sense of sections 1.1.3 and 1.1.4.

In order to allow a detailed control over sequencing and the deliberate use of side effects, it has been made possible to prevent any replacement or rearrangement of this sort. This is done by using the note sic at the head of the block. This has the effect of preventing the replacement or rearrangement of the commands inside the block. (See Section 9.8.2)
1.2 **Syntactic Problems**

1.2.1 **Publication and Canonical CPL**

Publication CPL contains a number of convenient programming devices. These are described in the Elementary Programming Manuals and a brief summary is given here in Section 2.1. Some of these features are not easily described in a formal system, and so publication CPL is formally defined by describing a transformation which changes a publication CPL document into canonical CPL. A description of this transformation is given in words in Section 2 and as a CPL program in Appendix 1. It should be noted that the transformation from publication to canonical CPL can be carried out by a number of processes and that the one described in this manual is by no means the only possible one.

Canonical CPL, which is introduced in this way as a device to aid in description of the language, is more easily subject to a formal description which occupies Part II of this report. A program in canonical CPL is a context-free character string with no dependence on layout or representation.

The existence of canonical CPL aids the task of specifying other hardware representations, such as those using cards, quite substantially. Logically unimportant changes, for example to the identifier rules, may be advisable in such cases. These alternatives are readily defined by specifying their transforms into canonical CPL.

Any practical implementation of CPL will probably accept an input which differs to some extent from both publication and canonical CPL. One possible way of constructing a compiler for such a system is to transform the locally defined input stream into a representation of canonical CPL before further processing. The program given in Appendix 1 would serve as such a PREPROCESSOR for a system whose local input language was publication CPL. Its inclusion may help to suggest a method for writing similar preprocessors for other local representations of CPL.

Publication CPL with only minor changes in the character set is used as the hardware representation at the London and Cambridge establishments.

1.2.2 **Syntax Rules**

Syntax descriptions of canonical CPL are given in terms of metalinguistic formulae. Sequences of letters enclosed in brackets \(< >\) represent syntactic categories; they are
chosen to be words describing approximately the meaning of 
the corresponding strings. The sign ::= separates the 
category being defined on its left from the defining 
sequence on its right. The vertical bar | is used to 
separate alternative sequences. Any sign which is not a 
category stands for itself. Juxtaposition of signs and/or 
categories signifies juxtaposition of the sequences they 
denote.

Metalinguistic brackets < > may be nested and thus used 
to group together more than one constituent sequence (which 
may contain alternatives). An integer subscript may be 
attached to a metalinguistic bracket and used to specify 
repetition; if it is an integer n, then the sequence within 
the bracket must be repeated at least n times; if the 
integer is followed by a - sign, then the sequence may be 
repeated at most n times or it may be absent.

For example, the formula

<ab> ::= <ab><d>|<ab>,-_1 [ ]

gives a recursive rule for the formation of values of the 
category <ab>; a legitimate value may be formed by some 
other value followed by a value of <d>, or by a (possibly 
absent) value followed by [, or by ]. If the values of 
<d> are the decimal digits, some values of <ab> are

[1]((137)
(12345)
(()
[6

Grouping or association rules can be expressed in two 
ways in the syntax. Unsubscripted recursive formulae or 
groups of formulae imply a corresponding semantic grouping. 
Subscripted formulae which are generally not recursive) 
implicy no such grouping. Thus:

<p> ::= <q>|<p><q

implies association to the left - i.e., a grouping of the 
type (((qq)q)q) and

<r> ::= <s>|<s><r

implies association to the right - i.e. (s(s(ss))) while

<x> ::= <y><z><y>

and

<x> ::= <<y><z>><y

have the same meaning and imply no association among the 
components of x so that the only grouping is (yzyzyz).

Syntax descriptions are given adjacent to those 
sections of text which describe the corresponding semantics,
and a complete syntax is given in Appendix 5.

For clarity, the $\phi$ separators are not included in the syntax formulae.

In order to avoid confusion, the canonical CPL symbols $<>\ll\gg$ and $|$ are replaced in syntactic formulae (where they might be confused with the metasyntactic symbols $<>|$) by the basic symbols $\textless\text{than}$, $\text{greater}\text{than}$, $\text{much}\text{less}\text{than}$, $\text{much}\text{greater}\text{than}$, $\text{bar}$.

1.2.3 Purposes and Limitations of Syntax

There is often some confusion about the precise meanings of the words syntax and semantics and some doubt as to where to draw the line between them. In this report we do not attempt either to discuss this issue or to be particularly nice in our distinction between the two; roughly speaking we call anything we express in the notation described in the last section 'syntax', and everything else in this manual 'semantics'.

The purpose of our syntactical analysis is to allow a CPL program to be divided into smaller segments, known as syntactic categories, in a way which will make the subsequent discussion of its effect (or meaning) simpler and more precise. Any text which cannot be analysed in this way will not be a correct CPL program, and this fact allows a compiler to detect many of the slips and trivial errors in a program at an early stage by a rather superficial 'syntactic' analysis. However, the converse is by no means true, as a text which is syntactically correct in the sense of this paragraph may be semantically meaningless.

The syntactic rules in this report are not even sufficient to group a CPL text completely. In some cases (e.g., infixed-expressions) they leave the final grouping to a later stage of the analysis which we have here included in the semantics. In others, the syntactic categories correspond not to groupings of the text but to semantically related items (e.g., relations).
1.3 Semantic Problems

1.3.1 General Approach

As no satisfactory and generally accepted method of describing the semantics of a programming language has yet emerged, the method adopted in this report is a mixture of informal description in English sometimes illustrated by examples in publication CPL with, in some cases, definitions in terms of other CPL forms. Some of the concepts used in these descriptions are discussed in Sections 1.3.2 to 1.3.5.

When giving examples in publication CPL it is convenient to be able to use variables whose values are individual members of various syntactic categories. Upper case Roman letters with numerical subscripts are used for this purpose. The letter is chosen to give some indication of which syntactic category is being represented, and the suffix is used identify the particular individual member of this category. Thus, for example, when discussing the assignment command which has the syntactic form

\[
\text{<assignment command> ::= <expression> ::= <expression list>}
\]

the example used is

\[
E_i := E_x
\]

which makes it possible for the subsequent discussion to refer to the components of the command as \(E_i\) and \(E_x\).

1.3.2 Data Items, Types

A CPL program is concerned with operations on and relations between certain objects known as DATA ITEMS, (or sometimes merely as ITEMS). These can be thought of in several ways. At one level they may consist of patterns of magnetization in a core store, at another they may be thought of as bit patterns or binary words. At another level they may be divided into 'instructions' and 'data', while at another they may be thought of as being numbers or letters.

The point of view adopted by CPL is that ultimately all data items are represented by bit patterns of various lengths. However, these bit pattern represent other, generally abstract objects, such as numbers or functions and their importance stems from this representation. This attitude allows us to talk about the abstract objects in CPL without being concerned with any particular representation.
as a bit-string (which may well be implementation dependent) but at the same time makes it possible to consider explicitly questions which involve this representation.

The TYPE of a data item determines its representation and at the same time constrains the range of entities which may be represented. Thus, for example, the type Integer can only be used to represent integers in a certain range. Both the extent of this range and the details of the representation of integers within this range are implementation dependent. The existence of the type Integer, however, is a property of the language.

CPL provides for a fairly wide range of types which are discussed in some detail in Section 3.2.

It is possible for a single abstract entity to be represented in more than one way as a bit pattern. The Integer 3, for example, can be a data item of type real or one of type Integer (among other possibilities). These two items would be quite distant and their bit-patterns would, in general, be different. There are a series of TRANSFER FUNCTIONS available in CPL which have the effect of changing from one form of representation to another without altering the abstract entity being represented. These are discussed further in Section 3.3.

The various types in CPL provide representations for a number of well-defined classes of abstract objects. It is these abstract objects which constitute the data items.

In some cases (for example, the type real) the representation of the abstract object is only approximate. The usual situation is that a range of entities all have the same representation. In these circumstances it is important to be clear whether the CPL program is discussing the ideal abstracts or their approximate representations. In other cases, however, where the representation is exact there is no significant difference in outcome whichever view is taken, and it is considerably simpler in these cases to regard CPL as manipulating the abstract entities directly and to leave the questions of representation to the implementation. It is partly for this reason that the rearrangement rules have been included in CPL.

This implies that in normal operation CPL is to be regarded as describing operations on abstract objects and not on their representations. In those cases where the difference between the approximation of the representation and the ideal abstract are of importance, it is possible to specify the exact sequence of elementary operations to be performed on the representations by using the note slc at the head of the block (see Sections 9.8.2 and 1.1.5).

In accordance with this view there are certain specific situations in which transfer functions are inserted automatically by the implementation in order to ensure that a correct representation is used.
1.3.3 *Transfer and Representation Functions*

Certain functions are available in CPL which deal explicitly with the representation of a data item. One class of these, known as TRANSFER FUNCTIONS, have as their aim the control of the type by which an item is represented. Another, known as REPRESENTATION FUNCTIONS, allow access to the actual bit patterns used in the representations; these, of course, are strongly implementation dependent.

Transfer functions are, in general, polymorphic; they take a single argument, which may be of any meaningful type, and produce a single result which has the same value as its argument (or an approximation to it) and has a specified type. These functions are listed in Section 3.3.1 and are available for explicit use by the programmer. They are also inserted automatically by the compiling system in certain places to ensure that a suitable representation is used.

While the finer details of the transfer functions (and in particular, their alarm conditions) are implementation dependent, their general nature, and the situations in which they are to be inserted automatically by the compiling system are a part of the language. These are described in detail in Section 3.3.

Representation functions are not concerned with the abstract value of their arguments, merely with the bit string which represents them in the current implementation. (see Note at end of this section.) They are not, in general, inserted by the compiling system and their use by a programmer serves as a warning that the program containing them is probably implementation dependent.

There is one exception to the statement that representation functions are not inserted by the compiling system. The ability to use table lookup as a method of processing non-numerical items (such as characters) is so convenient and widely used that the fact that it involves representation explicitly is often overlooked. In order to preserve this facility, the representation functions which take single character strings, logicals and index into each other may be invoked automatically by the compiling system. The details of this, and of the other representation functions are given in Section 3.3.

**NOTE:** In this section and elsewhere where bit strings and items of type logical and long logical are discussed, it has been tacitly assumed that the implementation will be on a binary machine. For implementations on a decimal machine, considerable revision of the functions and operations dealing with these types would be necessary.
2. The Transformation from Publication to Canonical CPL

2.1 Features of Publication CPL
   2.1.1 General Principles
   2.1.2 Terminators and Layout
   2.1.3 Brackets
   2.1.4 Conditional Expressions
   2.1.5 Other features

2.2 Categories Recognized During Transformation
   2.2.1 Names
   2.2.2 Left Section Bracket
   2.2.3 Right Section Bracket
   2.2.4 Strings
   2.2.5 Dot string
   2.2.6 Numbers

2.3 Rules for Transformation

NOTE  This section is taken from an earlier version and has not been edited. It may well need major revision and is only included to give the reader some idea of the chief features of publication CPL.
2.1 Features of Publication CPL

2.1.1 General Principles

The general principle underlying publication CPL has been to relieve the user, where possible, of the labour of including redundant information in his program. To the user, a program is best regarded as a document written in lines on sheets of paper and, as in studying the content of other documents, its layout may well convey information in the most convenient form. The publication language, therefore, makes use of abbreviations, permits omissions where no ambiguity is possible, and makes use of the layout to convey part of the meaning.

A programming language is more readily described in a formal way if programs are regarded as context-free strings of symbols. Canonical CPL is such a language, and the transformation from publication to canonical is here described. The verbal descriptions in this section describe its main features in an informal way. A more rigorous definition of the transformation is given as a program in publication CPL in Appendix 1.

2.1.2 Terminators and Layout

A command or definition may be terminated in one of three ways:

1) Explicitly, by use of a semicolon.

2) By its layout; an end of line is a terminator unless the end of line occurs at a point in the program where the context indicates that a command could not terminate at that point. Ends of lines which cannot be accepted as terminators or which are adjacent to the symbol \texttt{c} are ignored.

3) Implicitly; a terminator may be omitted if its presence is not essential to avoid ambiguity.

The use of "space" in CPL is derived from its use in written language in general. Spaces may be inserted freely between words to improve the layout. They are not used within words, and they must be used to separate words whose juxtaposition might cause ambiguity.

For example, consider the commands

\[
test \ x = y \ then \ z := z + \texttt{lerz} := l + z \ (a + b); \ Total := \text{Present Factor}
\]

The semicolon before "or" may be omitted. The end of line following "+" cannot terminate a command and is therefore ignored. A space is necessary only between "Present" and "Factor" to avoid ambiguity (this is a case of implicit multiplication).
2.1.3 Brackets

It is often the case in writing programs that several nested sections of program come to an end at the same point, where a string of closing section brackets has to be inserted. This is an error-prone operation as the number of closing brackets needed is easily miscounted.

Section brackets in CPL may therefore be distinguished from each other by attaching names or tags to them. A closing section bracket corresponds to the nearest opening bracket earlier in the program which bears the same tag. It defines the end of the section thus specified, and it implies the closure at this point of any contained sections of program which have not yet been closed.

Examples of the use of tagged section brackets may be found in the Introductory Manual and in Appendix 1.

2.1.4 Conditional Expressions

In publication CPL, the form of a conditional expression is

\[ E_1 \rightarrow E_2, E_3 \]

Any occurrence of a comma used in a conditional expression is replaced in the canonical form by the basic symbol "comma".

2.1.5 Other features

Monadic occurrences of + and - are allowed in publication CPL. In the canonical form, they are replaced by the basic symbols "pos" and "neg".

A wide range of synonyms is available for the basic underlined words in the canonical form. Many examples are found in the Introductory Manual which contains a list of synonyms.

Comments may be inserted freely in publication CPL text. A comment is introduced by a double bar and continues up to the end of that line; the whole of this text is ignored.

In some cases, symbols may be omitted in the publication form if no ambiguity is caused; for example,

```cpp
if B then goto L
```

may be written

```cpp
if B goto L
```
2.2 Categories recognised during Transformation

All instances of the following six categories are recognised during the transformation process. Their descriptions, and the modifications introduced during transformation, are described here. Note that strings of characters or primes mentioned in this section may be null, members.

2.2.1 < name >

A single lower case letter followed by any number of primes, or an upper case letter followed by a tag.

A tag is a string of letters of either case, digits and dots, followed by any number of primes.

In the canonical form, the symbol \( \notin \) is inserted before all occurrences of names.

2.2.2 < left section bracket >

The symbol \$ followed by a tag. The canonical form is unchanged.

2.2.3 < right section bracket >

The symbol \$ followed by a tag. The canonical form is unchanged.

Note that two section brackets with identical tags are said to "match".

The closing brackets omitted in publication CPL as in 2.1.3 above are inserted by the transformation.

2.2.4 < string constant >

A string of characters starting and ending with a prime. Within a string constant the symbol 10 (subscript ten) is used as an escape character:

\[ 1010 \text{ stands for } 10 \]
\[ 10' \text{ stands for } 1 \]
and the meaning of any other character following a single is implementa-
tion dependent.

In the canonical form, the symbol $\#$ is inserted before all instances of string constants.

2.2.5 $<$dot string$>$

A string of dots and spaces which must include at least 2 dots and which is surrounded by commas. The canonical form is as follows:

\[\ldots\ldots\]

2.2.6 $<$number$>$

A string of any number of digits and dots which may include spaces and must include at least one digit. The canonical form is preceded by the symbol $\#$. 

2.3 Rules for Transformation

These rules are intended as a brief verbal guide to the operation of the preprocessor program given in Appendix 1. They describe the operations performed on a complete publication text in a series of four passes, the end product being the corresponding canonical text. The operations described under each pass are done together.

Pass 1

1. Remove all comment text.

Pass 2

2. Recognise all instances of the six categories described in 2.2 above, and transform where necessary as described in that section.

Pass 3

3. Insert matching closing section brackets where required.

4. Instances of commas used in conditional expressions are replaced by "comma".

5. Change all new lines to semicolons, replace consecutive semicolons by a single semicolon. Consider the symbols adjacent to each semicolon. If either is \( c \), remove the semicolon. Unless the preceding symbol can legally precede a semicolon and the succeeding symbol can start a command, delete the semicolon.

Pass 4

6. Remove all occurrences of the basic symbol \( c \).

7. Insert \textit{do} between any pair of symbols such that the first of these cannot precede a command and the second must start a command.

8. Recognise monadic uses of \( + \) and \( - \) and replace them by \textit{pos} and \textit{neg} respectively.
3 PRELIMINARIES

3.1 Canonical Form
   3.1.1 General
   3.1.2 CPL Publication Alphabet
   3.1.3 Basic Symbols
   3.1.4 Basic Categories

3.2 Types
   3.2.1 General
   3.2.2 Numerical Types
   3.2.3 Logical Types
   3.2.4 Other Types

3.3 Transfer and Representation Functions
   3.3.1 Programmers Transfer Functions
   3.3.2 Basic Transfer Functions
   3.3.3 Automatic Insertion of Transfer Functions
   3.3.4 Polymorphic Operators
   3.3.5 Representation Functions

3.4 Constants
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   3.4.2 Syntax
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Note The Type character will probably be added. This will involve some alteration to strings as possibly elsewhere. Compound Data Structures will probably also extend and alter the whole concept of Types so that this section may have to be modified considerably or extended.
3.1 Canonical Form

3.1.1 General

A program in canonical CPL consists of a string of items which are either BASIC SYMBOLS or members of a BASIC CATEGORY. A basic symbol is a single character from the CPL basic alphabet $\mathcal{A}_B$ defined in Section 3.1.3. A member of a basic category consists of a character from $\mathcal{A}_B$ identifying the category followed by a word (string of characters) from the possibly implementation dependent alphabet associated with the category. The categories and their associated alphabets are described in Section 3.1.4.

Every implementation must have a method of representing all the characters in $\mathcal{A}_B$ and the alphabets associated with the basic categories by one or more characters from its IMPLEMENTATION ALPHABET $\mathcal{A}_I$. These are the characters which occupy one print position in the program together with certain extra symbols representing the layout.

The implementation characters may in turn be represented by one or more HARDWARE CHARACTERS from the alphabet $\mathcal{A}_H$. These are the basic hardware units of input and output such as a single row of punched tape, a single column of a punched card or a single keystroke on a directly connected keyboard.

Thus, for example, in a hardware implementation which uses a backspace to provide compound or overstruck characters, the following three strings of length six each produces the same printed result (The symbol $\dagger$ is used temporarily to denote the hardware character 'backspace').

```
\dagger \_f\dagger _
\dagger \_f\dagger _
\_\dagger \_f
```

In each case the result produced should be the two character string from the implementation alphabet $\mathcal{A}_I$. This in turn represents the single character (basic symbol) $\mathcal{I}f$ from the alphabet $\mathcal{A}_B$.

The transformation from a string of hardware characters to a string of implementation characters is largely implementation dependent. However, it should be arranged so that hardware character strings which would produce the same printed image will produce the same implementation character string as illustrated by the example given above.

The transformation from a string of implementation characters to canonical CPL is done by a preprocessor of the type described in Section 2 and Appendix 1.
3.1.2 CPL Publication Alphabet

There is one 'Implementation' alphabet which is not implementation dependent; this is the publication alphabet \( \mathcal{A}_p \) which contains the following 162 printing characters:

\[
\begin{align*}
A & B C D E F G H I J K L M N O P Q R S T U V W X Y Z \\
\text{a b c d e f g h i j k l m n o p q r s t u v w x y z} & 0 1 2 3 4 5 6 7 8 9 \\
\end{align*}
\]

and the two non-printing characters 'space' and 'newline'.

These characters are used in the rest of the manual except that the three symbols \( \ll \geq \leq \) may be typed so as to occupy two print positions each.

3.1.3 Basic Symbols

The CPL Basic Alphabet \( \mathcal{A}_B \) contains the 109 symbols listed below. The symbols are given in the representation used in the syntax tables and also, if necessary, the form or forms by which they can be represented in the publication language. It is a general rule of the publication language that when representing a basic symbol by an UNDERLINED WORD (including words of one letter) no distinction is made between upper and lower case letters and spaces, whether underlined or not, are ignored. Thus the two strings

\text{GO TO} \quad \text{go to}

in the publication language both represent the same basic symbol which is represented in the table below as goto.

(a) The following 25 symbols from \( \mathcal{A}_p \) also represent unique basic symbols from \( \mathcal{A}_B \):

\[
\begin{align*}
\times / & \uparrow \downarrow = \neq \geq \leq \wedge \vee \equiv \neq \sim \approx \\
; & : = ( ) \{ \} \_ \$ \rightarrow \rightarrow \to \end{align*}
\]
(b) the following five basic symbols are represented in the syntax tables by underlined words and in the publication language by single symbols. (see Section 1.2.2)

<table>
<thead>
<tr>
<th>Syntax Form</th>
<th>Publication Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>bar</td>
<td></td>
</tr>
<tr>
<td>greaterthan</td>
<td>&gt;</td>
</tr>
<tr>
<td>lessthan</td>
<td>&lt;</td>
</tr>
<tr>
<td>muchgreaterthan</td>
<td>&gt;&gt;</td>
</tr>
<tr>
<td>muchlessthan</td>
<td>&lt;&lt;</td>
</tr>
</tbody>
</table>

(c) The following 54 basic symbols are represented in the syntax tables and in the publication by the same underlined word.

| and               | general         | n          | test      |
| array             | goto            | note       | test      |
| Boolean           | l               | prefer     | true      |
| break             | if              | r          | type      |
| cpx               | index           | real       | unless    |
|                   | integer         | repeat     | until     |
| d                 | repeatuntil     |                   | update     |
| dcpx              | l               | repeatwhile | vector    |
| double            | label           | resultis   |           |
| doublecomplex     | let             | return     |           |
|                   | load            |             |           |
| false             | logical         | sic        | x         |
| finish            | longlogical     | step       |           |
| fix               | matrix          |             |           |
| fixed             |                 |             |           |
| for               |                 |             |           |
| free              |                 |             |           |

(d) The following 14 basic symbols can be represented in the publication language by any one of a number of synonyms. In the table below, the first form is that used in the syntax tables.

| allbe             | bothare         | bothbe        | are       |
| be               | is              |               |           |
| constant          | const           | then          | thendo    |
| do               |                 |               |           |
| forexternal       | forext          | fn            |           |
| function          |                 | ordo          |           |
| or               |                 | rec           |           |
| recursive         |                 |               |           |
(e) The following six basic symbols are represented by only three symbols in the publication language. The preprocessor decides from the context which is the appropriate basic symbol to use.

<table>
<thead>
<tr>
<th>Syntax form</th>
<th>Publication Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>comma</td>
<td>,</td>
</tr>
<tr>
<td>pos</td>
<td>+</td>
</tr>
<tr>
<td>neg</td>
<td>-</td>
</tr>
<tr>
<td>der</td>
<td>+</td>
</tr>
<tr>
<td>val</td>
<td>-</td>
</tr>
<tr>
<td>valof</td>
<td>-</td>
</tr>
<tr>
<td>routine</td>
<td>rt</td>
</tr>
<tr>
<td>reference</td>
<td>ref</td>
</tr>
<tr>
<td>referenceof</td>
<td>refof</td>
</tr>
<tr>
<td>variable</td>
<td>var</td>
</tr>
</tbody>
</table>

(f) The remaining five basic symbols are the following:

- **dotstring** which has the publication language representation of at least \( \ast \) dots, possibly interspersed with spaces and surrounded by commas. The following are three examples of **dotstrings** in the publication language:

\[
\ldots, \ldots, \ldots, \\
\hat{n}, \hat{d}, \hat{s}
\]

which are used to indicate the basic categories name, number and string-constant respectively.

- \( \phi \) which is sometimes used when writing canonical CPL strings to separate the basic symbols where there might otherwise be confusion. It has no other significance.
3.1.4 Basic Categories

The precise definition of the basic categories is implementation dependent. The definitions given in this section apply to the publication language. The basic categories are recognised by the preprocessor.

(a) Names

A member of the basic category <name> in canonical CPL consists of a name-word from the alphabet \( \mathcal{A}_N \) preceded by the basic symbol _ and followed by the basic symbol \( \phi \) (both from the alphabet \( \mathcal{A}_A \)).

For publication CPL the alphabet \( \mathcal{A}_N \) contains the following 64 symbols:

\[
\begin{align*}
\text{A} & \quad \text{B} & \quad \text{C} & \quad \text{D} & \quad \text{E} & \quad \text{F} & \quad \text{G} & \quad \text{H} & \quad \text{I} & \quad \text{J} & \quad \text{K} & \quad \text{L} & \quad \text{M} & \quad \text{N} & \quad \text{O} & \quad \text{P} & \quad \text{Q} & \quad \text{R} & \quad \text{S} & \quad \text{T} & \quad \text{U} & \quad \text{V} & \quad \text{W} & \quad \text{X} & \quad \text{Y} & \quad \text{Z} \\
\text{a} & \quad \text{b} & \quad \text{c} & \quad \text{d} & \quad \text{e} & \quad \text{f} & \quad \text{g} & \quad \text{h} & \quad \text{i} & \quad \text{j} & \quad \text{k} & \quad \text{l} & \quad \text{m} & \quad \text{n} & \quad \text{o} & \quad \text{p} & \quad \text{q} & \quad \text{r} & \quad \text{s} & \quad \text{t} & \quad \text{u} & \quad \text{v} & \quad \text{w} & \quad \text{x} & \quad \text{y} & \quad \text{z} \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 & \quad 9
\end{align*}
\]

A name-word consists of a single lower case letter followed by any number of primes, or an upper case letter followed by a tag.

A tag consists of a string of letters (of either case), digits or dots of any length followed by any number of primes. A tag may not include spaces - indeed a space is not a symbol in the alphabet \( \mathcal{A}_N \).

The alphabet \( \mathcal{A}_N \) is a sub-alphabet of \( \mathcal{A}_F \) and its symbols are represented in the publication language by the same symbols from \( \mathcal{A}_F \).

(b) Numbers

A member of the basic category <number> in canonical CPL consists of a number-word from the alphabet \( \mathcal{A}_D \) preceded by the basic symbol _ and followed by the basic symbol \( \phi \) (both from the alphabet \( \mathcal{A}_A \)).

For publication CPL the alphabet \( \mathcal{A}_D \) contains the following 11 symbols:

\[
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9
\]

A number-word consists of any string of these symbols containing at least one digit.

The alphabet \( \mathcal{A}_D \) is a sub-alphabet of \( \mathcal{A}_F \) and its symbols are represented in the publication language by the same symbols from \( \mathcal{A}_F \).

(c) String-Constants

A member of the basic category <string constant> in canonical CPL consists of a string-word from the QUOTABLE ALPHABET \( \mathcal{A}_Q \) preceded by the basic symbol _ and followed by
the basic symbol \( \phi \) (both from the alphabet \( \mathcal{A} \)).

For publication CPL the alphabet \( \mathcal{A} \) contains the 162 printing characters from the publication alphabet \( \mathcal{P} \) (see Section 3.1.2) together with the three non-printing characters:

'space' 'backspace' 'newline'

A string-word consists of any number of these symbols (including zero).

In publication CPL a string-constant is distinguished from other parts of the text by being enclosed by primes. This fact, together with the rules for dealing with comments in the preprocessor mean that although the characters of \( \mathcal{A} \) and \( \mathcal{P} \) are very similar, some of the characters from \( \mathcal{A} \) cannot be straightforwardly represented by the same character from \( \mathcal{P} \). The precise rule is the following:

The following five characters from \( \mathcal{A} \) have a special representation in characters from \( \mathcal{P} \) as shown:

<table>
<thead>
<tr>
<th>Character from ( \mathcal{A} )</th>
<th>Representation in ( \mathcal{P} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>backspace</td>
<td>'b'</td>
</tr>
<tr>
<td>newline</td>
<td>'n'</td>
</tr>
<tr>
<td>'</td>
<td>':'</td>
</tr>
<tr>
<td>'</td>
<td>'!'</td>
</tr>
<tr>
<td>'</td>
<td>'ll'</td>
</tr>
</tbody>
</table>

The remaining 160 characters in \( \mathcal{A} \) are represented by the same character from \( \mathcal{P} \). In addition, the non-printing character 'space' in \( \mathcal{A} \) may also be represented by the characters 's' from \( \mathcal{P} \).

Note that string-constants are defined above by the way in which the characters from \( \mathcal{A} \) are represented, and not in the way in which an arbitrary string of characters from \( \mathcal{P} \) can be interpreted. This implies that not all possible strings enclosed in string-quotes (primes) are acceptable. In particular, the ESCAPE CHARACTER ' is only defined if followed by one of the six characters b n ' ' ' or s.

Publication CPL has the additional rule (implemented by the preprocessor) that new lines and trailing spaces (at the right end of a line) are ignored in string-constants.

(d) Section-Brackets

In publication CPL the basic symbols \( \$ \) and \( \$ \) may be followed by a tag taken from the alphabet \( \mathcal{N} \) (see (a) above). These are used by the preprocessor to insert extra closing section brackets (\( \$ \)) where necessary so that in a canonical CPL program all section brackets nest and match correctly. This means that the tags associated with them in the publication language are no longer necessary so that they do not form a part of the canonical CPL program.

Thus section brackets do not form a basic category in the strict sense. They must, however, be treated as such in
the preprocessor and many implementations will find it desirable to continue to associate a tag with each section bracket even when it is no longer logically necessary in order to give some assistance in locating errors in the program.
3.2 **Types**

3.2.1 **General**

The following sections list all the types of data item currently included in CPL and give a brief description of them. The details are implementation dependent.

Further types may be added from time to time, and it is possible that the incorporation of a more comprehensive treatment of compound data structures may force a considerable revision or extension of the concepts of this section.

The rules concerning transfer functions are collected in Section 3.3 for convenience. They are discussed in more general terms in Sections 1.3.2 and 1.3.3.

3.2.2 **Numerical Types**

**real**

A real number, whose range and precision are implementation dependent.

**integer**

An integer, whose range is implementation dependent.

**index**

An integer, whose range is probably less than that of integer and whose use may improve object program efficiency, for example, in subscripts.

**complex**

A pair of real numbers taken in order as the real and imaginary parts of a complex number.

**double**

A real number with approximately double the precision of type real.

**doublecomplex**

A pair of real numbers taken in order as the real and imaginary parts of a complex number. The components have approximately double the precision of the components of a complex data item.
3.2.3 Logical Types

**Boolean**
A truth value; its value is either **true** or **false**.

**logical**
A bit string of a fixed and implementation dependent length.

**longlogical**
A bit string of a fixed and implementation dependent length probably greater than that of **logical**.

The intention of **logical** and **longlogical** is that it should be possible to transform reasonably large sections of type **index** into **logical** and of type **real** into **longlogical** without loss of information.

In an implementation based on a binary word oriented computer where **reals** occupied one word and **index** a shorter, possibly address-length, segment, **logicals** would probably be the length of an Index register and **longlogicals** a whole computer word.

3.2.4 Other Types

**string**
A string of characters of any length including null (See Sections 3.1.4, 3.4.5 and 3.4.6).

**label**
A location in the program and a description of an environment of that area of program (See Section 9.3).

**function**
A representation of a function (See Section 10).

**routine**
A representation of a routine (See Section 10).

**array**
A representation of an array (See Section 12).

**type**
A data item whose value is a data item type.

**general**
A data item whose type may vary dynamically; it may be a data-type, array-type or function-type but not of a single-type-list (See Section 7.1).
3.3 Transfer and Representation Functions

3.3.1 Programmers Transfer Functions

The following transfer functions are available to the programmer:

- DoubleComplex  dc
- Complex        c
- Double         d
- Real           r
- Integer        n
- Index          x
- LongLogical    ll
- Logical        l
- String         s
- Boolean        b

Each takes a single argument which may be of any meaningful type and produces as its single result an item of the type indicated by its name. Each function exists in two forms, according to the mode of the context in which it is used.

a) The R-mode Transfer Functions

These take a single R-mode argument and produce a single R-mode result. The details are implementation dependent, but the results can be described with the aid of the following table which uses the abbreviations indicated above for types.
### Table of Initial Transfer Functions

The entry corresponding to a pair of types in this table indicates the first type to which the data item is to be transformed in a transfer from the first type to the second. Thus the transfer from `string` to `doublecomplex` would pass through the sequence.

```
string → index → integer → real → complex → doublecomplex
```

Further details of the BASIC TRANSFER FUNCTIONS which make up chains of this sort are given in Section 3.3.2.

The transfers indicated by entries enclosed in parenthesis can only be initiated by an explicitly written transfer function. The others may, in suitable circumstances, be invoked automatically by the compiling system (see Section 3.3.3).

There are direct transfer functions from every type (including those not mentioned in the table) to type `general` which make no change in the representation. Transfers from `general` to other types can only take place in circumstances when a transfer from the dynamically current type associated with the `general` would be permitted.

b) The L-mode Transfer Functions

These are `load-update-pairs` and can be defined in terms of the R-mode functions. Thus, for example, the L-mode
transfer function which takes real into complex is equivalent to the following.

Function RealToComplex [ref real x] be
    load $ result is Complex [$ x $]
    update $ x := Real [$ rhs $]

c) Mode Forcing Functions
The functions
    RValue [ x ]
    LValue [ x ]
which can take an argument of any type, force the evaluation of their argument in the mode indicated. They are not true transfer functions, but may invoke the mode transfers described in Section 4.2. These functions may be of use when it is desirable to specify the mode of evaluation in circumstances where the context would not otherwise do this (e.g. the argument list for an application of a variable or formal functions).

d) Other Types
There are no transfer functions except those mentioned above. Representation functions are discussed in Section 3.3.5.

3.3.2 Basic Transfer Functions

a) index to integer
    integer to real
    real to double
    real to complex
    double to double complex
    complex to double complex
These are standard numerical transfers with changes of representation but normally no loss of information. With the possible exception of integer to real no alarm conditions should be necessary.

b) integer to index
    double to real
    double complex to complex
These involve a decrease in precision (and possibly of range) without a change of the nature of the number. They will normally involve a rounding or truncating operation and should give an alarm if the range is exceeded.

c) real to integer
    complex to real
    double complex to double
These involve a change in the nature of the number probably without alteration of the precision or range. The
two complex to real transformations should show an alarm if the imaginary part of the complex number is not negligible in an implementation dependent sense compared with the real part. The real to integer transfer is intended to allow operations on integers to be carried out using reals even if the representation of reals does not allow exact integer arithmetic to be performed. The transformation should therefore show an alarm whenever the divergence of the real from an integral value is more than a certain implementation dependent amount. It is open to the implementation to make this quantity anything between 0 and 0.5 so that programs which require the nearest integer to a real whose exact value should not be integral should use the basic function Round (see Appendix 3).

An alarm should also be given if the result is out of range.

d) logical to longlogical
   longlogical to logical
   The logical is taken to be the right hand (least significant) end of a longlogical whose remaining bits are all zeros. As these transfer functions may be inserted automatically, there should be an alarm on an attempt to transform a longlogical whose extra bits were not all zeros. The basic function Mask (see Appendix 3) may be used to ensure this if required.

e) logical to index
   Index to logical
   string to index
   Index to string
   logical to string
   string to logical
   Boolean to index
   index to Boolean

   These transfers are made by considering all the types concerned to be integers. For logicals this is done by treating them as unsigned (positive) binary integers. For strings the functions are only defined for strings of length 1 (single characters); for these, the corresponding integer is that given in the collating table (see Section 3.4.6). If several characters have the same number in this table, one of these should be nominated for use by the transfer functions. For Booleans the correspondence is True ↔ 1 and false ↔ 0.

An alarm should be given if the result is out of range or, in the case of transfers to string, if no suitable character exists.
3.3.3 Automatic Insertion of Transfer Functions

The transfer functions corresponding to the unparenthesised entries in the table of Section 3.3.1, those to and from type general, and the -mode transfer functions (see Section 4.2) and no others, will be inserted by the compiling system as necessary in the following situations.

(a) When the type and/or mode of an expression is constrained by its context. This arises in the following cases.
   (i) The right hand side of an assignment statement. The mode is R-mode and the type must be that of the L-value it is to update (see Section 9.2).
   (ii) An argument of a function or routine call in the special case where the written operator is the name of a function or routine data item with the attribute constant, and whose formal parameter types and modes are explicitly (or by default) stated in the definition (see Sections 5.3.2 and 10.1.2).
   (iii) An argument of a polymorphic function, routine or operator after the particular version of the function, routine or operator has been determined by the rules given in Section 3.3.4, Appendix 3 or elsewhere.

(Note that programmers' functions and routines come under case (i) and basic and library functions under case (ii) or (iii).)

In these situations the compiler will insert the necessary transfer functions to match the mode and type of the expression to its context. If the necessary transfer functions either do not exist or are not allowed to be inserted automatically, the outcome is undefined.

(b) When the value of an expression may be that of one of a number of expressions. The choice being determined dynamically. This arises in the following cases:
   (i) The arms of a conditional-expression.
   (ii) When a block-expression contains two or more resultis commands.

In these situations each of the alternative expressions is transformed to the type which is the least upper bound of their individual types. This is defined in an obvious way from the diagram on the next page.
Hierarchy of Types

If the required transfer functions do not exist or cannot be inserted automatically, the least upper bound is taken as general.

Mode transfers are not required in these situations.

3.3.4 Polymorphic Operators

Polymorphic operators are operators which exist in several versions, all known by the same name. The choice between these versions is dependent on the types of their arguments. There are two classes of polymorphic operators, those for which a unique version exists for each allowable set of argument types, and those for which only a limited set of versions exist and transfer functions are used to extend the permissible set of argument types. Transfer representation functions are in the first class, most basic
functions and infixed operators are in the second. Polymorphic operators are a part of the language and no mechanism exists at present to allow a programmer to define his own polymorphic functions or routines.

The selection of the correct version of a polymorphic operator of the second class is governed by one of the following rules; the particular rule to be used being determined by the name (or in the case of an infixed-operator, the symbol) of the operator.

If any of the operands are of type **general** then this rule must be applied dynamically (i.e. at the time at which the operator is to be applied to its operands); the current types of the **general** arguments must be used to select the correct version of the operator as described below. If the type of the result depends on the version of the operator used in any way, the result must be converted back to type **general**. If all versions of the operator produce the same type (as occurs, for example, with relations where the result is always **Boolean**), then this is also the type of the result.

In cases where none of the operands are **general**, the rule selected can be applied by the compiling system and any transfer functions required may be inserted statically (provided this is allowed under the rules given in Section 3.3.3).

**Rule A**

The version of the operator selected is the least upper bound (in the sense of Section 3.3.3) of the types of all the operands and of the lowest available operator type. If no version of the operator of this type exists, or if the least upper bound is **general**, the effect is undefined. If a suitable operator exists, its type will define the types required of the operands all of which must be transformed to this type before the operator is applied. The type of the result is also determined by the version of the operator; in general it is either the same as that of the operands, or restricted to a single type such as **Boolean**.

**Rule B**

The operator requires all its parameters except one to be of a unique specified type, so that polymorphism occurs for only one of its arguments. The version of the operator is determined from the actual type of its only polymorphic argument as follows:

i) If a version of the operator exists which accepts an argument of exactly this type, this is the version chosen.

ii) If no such version exists, then the operator is taken to be a single specified version.

In the first case transfer functions may be inserted automatically for those arguments for which the operators is not polymorphic; in the second case they may be inserted for all arguments.
3.3.5 Representation Functions

The following representation functions are available to the programmer.

1. **Bitpattern[x]**

   This is a polymorphic function of the first class (see Section 3.3.4) which has the following versions:

<table>
<thead>
<tr>
<th>Argument Type</th>
<th>Result Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>real</td>
<td>longlogical</td>
</tr>
<tr>
<td>integer</td>
<td>longlogical</td>
</tr>
<tr>
<td>index</td>
<td>logical</td>
</tr>
<tr>
<td>character</td>
<td>logical</td>
</tr>
</tbody>
</table>

   Other implementation dependent versions may also exist. The effect is to treat the actual (implementation dependent) representation of its argument as a bit pattern of the corresponding type, any spare bits in the result type being filled with zeros. No transfer functions are inserted automatically for this function.

2. **Formreal[1]**  
   **Forminteger[1]**  
   **Formindex[1]**  
   **Formcharacter[1]**

   These functions, which are not polymorphic, together form the inverse of Bitpattern x. They take a longlogical or logical argument as indicated and treat it as if it were the internal representation of the type suggested by the name of the function.

   Other representation functions forming the inverse of further versions of Bitpattern may also exist.
3.4 Constants

3.4.1 General

Written constants have an R-value which is invariant and independent of any definitions or assignments made in the program. Their natural mode of evaluation (see Section 4.2) is R-mode so that they behave as if they had no L-value.

There are two forms of written constants: ATOMIC CONSTANTS which are usually basic symbols such as True or a single digit, and which have the property that their R-value depends only on their complete written form and cannot be deduced from the R-values of any of its components (if it can be decomposed at all); and COMPOUND CONSTANTS whose R-value can be deduced from the R-values of its components. A number such as 231 is a compound constant and its value can be deduced from the values of its component digits 2, 3 and 1.

Compound constants are a form of constant expression but the rules governing their evaluation are not those governing the other expressions in CPL.

As constants have an R-value which is immediately determinable from their written form, the concepts of scope and extent (see Section 8.2) are not applicable to them. In this respect they differ from named data items which have been defined with the attribute constant (see Section 7.3 and 10.3.2).

3.4.2 Syntax

<basic numerical constant> ::= <number><\<|\< |<\>\>\>\>\>\>\>\>
<constant prefix> ::= x | n | r | d | cpd | dcpd
<numerical constant> ::= <constant prefix><
<basic numerical constant>| 1

logical prefix> ::= 2 | 3 | 2l | 3l | 2h | 3h | 28 | 38

logical constant> ::= <logical prefix><number>| \bar | <logical prefix><number> \bar <logical prefix><number>

Boolean constant> ::= true | false
(constant) ::= (numerical constant)|logical constant)|
    (string constant)|Boolean constant)|
    (single type)

3.4.3 Numerical Constants

A basic-numerical-constant of the form

\[ N_1 \cdot 10 \cdot N_2 \]

where \( N_1 \) and \( N_2 \) are members of the basic category number, is
only defined if \( N_1 \) contains at most one dot and \( N_2 \) contains
no dots. The dot in \( N_1 \), if present, is taken as a decimal
point; if \( N_1 \) contains no dot it is taken as a decimal
integer. The \( 10 \) denotes that the \( N_2 \) following it is to be
interpreted as a decimal exponent. If there is a + or a -
between the \( 10 \) and \( N_2 \), this is taken as the sign of the
exponent; if there is no sign, the exponent is considered
positive.

Two unprefixed basic-numerical-constants which have the
same numerical values will always be represented in the same
way. Thus the constants

\[ 1, +1, 1.0, +0.1, 10 +1 \]

are interchangeable and only differ in the way in which they
are written on the paper.

The type of a numerical-constant prefixed by \( k, n, r, d, cpx, \) or \( dcpx \) is
(index, integer, real, double, complex, or
doublecomplex), respectively.

The type of an unprefixed numerical-constant is the
least upper bound (in the sense of Section 3.3.3) of its
PRESUMPTIVE TYPE and its MINIMUM TYPE.

The presumptive type is determined by context: if the
numerical constant stands alone on the right hand side of a
definition, the presumptive type is the preferred type if
this is numerical. In all other cases and contexts the
presumptive type is index.

The minimum type of a numerical-constant is determined
by its value. If this is integral and within the index
range, then the minimum type is index. If the value is
integral and outside the index range but within the integer
range, then the minimum type is integer. In all other cases
the minimum type is real.

The basic symbol \( i \) represents a numerical constant of
type complex whose value is a complex number with zero real
part and unit imaginary part. Note that with this exception
there are no unprefixed numerical-constants of type double,
complex, or doublecomplex, but that an expression such as
2 + 31

will be of type complex by virtue of the rules governing the types of infixed-expressions (see Section 6).

3.4.4 Logical Constants

The occurrence of 1 or \_ in a logical prefix indicates that the constant is to be represented in the type longlogical. Otherwise it is logical. If the prefix contains 2, the constant is a binary pattern and may only contain digits 0 and 1. If the prefix contains 8, it is an octal pattern and may only contain digits 0 to 7. Logical constants are assumed right justified unless bar is present, in which case justification takes place towards the side on which bar appears.

3.4.5 String-Constants

A string-constant is a member of a basic category and, as such, its syntax is not a part of canonical CPL (see Section 3.1.4).

In Publication CPL in particular a string-constant is identified by being preceded and followed by a prime. Inside these string quotes the character \_ is used as a single-character-quote or escape indicator. If it is immediately followed by one of the characters

\[ b \ n \ s \ ' \! \| \]

the pair is taken as the single string character (i.e., the single character from \( \mathcal{X}_Q \))

\[ \text{backspace newline space ' ! } \| \]

respectively.

Thus, for example, a string-constant in Publication CPL which was written as

'\_\_b\_\_b/'

has a length of 10 string characters

\[ s \text{ backspace } ' \_f' \text{ o backspace }/ \]

and when printed would produce
3.4.6 Character Representation

The internal representation of strings is by characters from the internal alphabet \( \mathcal{A}_x \). The number of characters in the alphabet is implementation dependent and the characters have a fixed association with (small) integers which is that used by the transfer functions between strings of length one (characters) and the types Index and Boolean. The characters of the alphabets \( \mathcal{A}_b, \mathcal{A}_x, \mathcal{A}_n, \mathcal{A}_v \) and \( \mathcal{A}_q \) may all be represented in \( \mathcal{A}_x \). The details of the representation are implementation dependent; in particular it is not necessary for the various alphabets to be represented by disjoint sections of \( \mathcal{A}_x \) nor is it necessary for single characters to be represented by single characters although this is very desirable at least for the quotable alphabet \( \mathcal{A}_q \). Tables showing the representation of \( \mathcal{A}_b \) and \( \mathcal{A}_q \) which are relevant to publication CPL are given in Appendix 2.

3.4.7 Other Constant Expressions

a) Boolean Constants
   The basic symbols \texttt{true} and \texttt{false} are constants of type \texttt{Boolean}.

b) Type Constants
   Any expression which falls into the syntactic category \texttt{<single type>} (see Section 7.1) is a constant of type \texttt{type}.

c) Other Types
   There are no constants of type \texttt{label}, \texttt{function}, \texttt{routine}, \texttt{array} or \texttt{general} except those introduced explicitly by a definition with the attribute \texttt{constant}. 

4 EXPRESSIONS

4.1 Syntax
4.2 Evaluation
4.3 Conditional-Expressions
  4.3.1 Syntax
  4.3.2 Semantics
4.4 Block-Expressions
  4.4.1 Syntax
  4.4.2 Semantics
4.5 Expression-Lists
4.6 Where-Clauses

4.1 Syntax

<prefixed operator> ::= <name>|<prefixed expression>| rhs | (
  <expression>)

<prefixed expression> ::= <prefixed operator>[[expression]]

<individual> ::= <constant>|<name>|<prefixed expression>|
  ~<indivdual>|<block expression>| rhs |
  pos | neg | (<expression>)

<relation> ::= muchlessthan | lessthan | ≤ | = | ≠ | ≥ |
  greaterthan | muchgreaterthan

<infixed operator> ::= + | - | * | / | ↑ | = | ≠ | ∧ | ∨ |
  ⇒ | <relation>

<infixed expression> ::= <<individual><infixed operator>1->0
  <individual>

<conditional expression> ::= <infixed expression>⇒
  <basic expression> comma <basic expression>

<block expression> ::= valueof <block>| referenceof <block>

<basic expression> ::= <infixed expression>|
  <conditional expression>

<expression list> ::= <basic expression>, <basic expression>

<expression> ::= <expression list>|
  <expression> where <in definition>
4.2 Evaluation

An expression basically consists of a name, a constant, or an operator with its operands; both the operator and the operands are themselves expressions. More generally an expression may be a sequence of such forms separated by commas.

An expression is a rule for the evaluation of a sequence of one or more quantities or values. Evaluation may in general be performed in either of two modes: L-mode and R-mode. The result of an expression evaluated in L-mode is a sequence of one or more L-values, and the result of an expression evaluated in R-mode is a sequence of one or more R-values (see Section 4.5). The number of members of such a sequence is normally restricted by the context of the expression. In particular, an expression which occurs as the operand of a monadic or infixed-operator (see Sections 5 and 6) must yield a single value.

The mode of evaluation of a particular expression is determined by context, although certain operators may have a natural mode of evaluation. For example, all monadic and infixed-operators require R-values as operands and produce single R-values as results, whereas prefixed-operators may take R-value or L-value operands and produce R-value or L-value results according to context. The natural mode of evaluation of a name is L-mode, and of a constant, R-mode. In both cases, the result is a single value.

In circumstances where the mode required by context conflicts with the natural mode, transfers are automatically effected as follows.

a) L-Value to R-Value

The data specified by the L-value is extracted to give an R-value.

b) R-Value to L-Value

A new (disjoint) L-value is created and the R-value becomes associated with it; the new L-value is given the attribute constant (see Section 7.3).
The following rules apply to the evaluation of operators and operands in expressions.

1. Except where otherwise stated, an operator and its operands (including their component parts) may be evaluated in any order or in parallel. An operator may be applied to its operands at any time after each of them has been evaluated.

2. Expressions may be transformed and re-ordered in any way that would give identical results if all evaluations were performed exactly.
4.3 Conditional-Expressions

4.3.1 Syntax

\[ <\text{conditional expression}> ::= <\text{infixed expression}> \rightarrow <\text{basic expression}> \text{comma} <\text{basic expression}> \]

In the conditional-expression

\[ E_1 \rightarrow E_2, E_3 \]

the syntax rules permit \( E_1 \) to be an infixed-expression and each of \( E_2 \) and \( E_3 \) to be a basic-expression. This implies that if either \( E_2 \) or \( E_3 \) is to represent an expression-list or is to be qualified by a where-clause, then this must be enclosed in parentheses; similarly, \( E_1 \) must be bracketed if it is itself a conditional-expression or is qualified by a where-clause.

4.3.2 Semantics

The conditional-expression

\[ E_1 \rightarrow E_2, E_3 \]

has the same value as the expression

\[ \text{reference of } \& \text{ test } E_1 \text{ then result is } E_2 \text{ or result is } E_3 \]

(see Sections 4.4 and 9.6) Note that on evaluating this conditional-expression \( E_1 \) is evaluated in R-mode and that one only of \( E_2 \) or \( E_3 \) is evaluated in L-mode. Thus the natural mode of evaluation (section 4.2) is L-mode.
4.4 Block-Expressions

4.4.1 Syntax

\[ \text{block expression} ::= \text{value of } \text{block} | \text{reference of } \text{block} \]

A block-expression enables the computation of one or more results to be written in the form of a command, or block. There are two forms depending on the required mode of evaluation.

- \text{value of } \text{block} \quad \text{which produces one or more R-values}
- \text{reference of } \text{block} \quad \text{which produces one or more L-values}

In either case, the block must contain one or more \textit{result is} commands (for syntax see Section 9.5.4). The type of each result of a block-expression is defined to be the least upper bound (as defined in Section 6.6.2) of the types of the corresponding members of the constituent expressions in all \textit{result is} commands which appear in the block, but excluding any which appear in further block-expressions within the block.

4.4.2 Semantics

A block-expression is evaluated as follows. The commands (and declarations, if any) are executed in sequence until one of the \textit{result is} commands is encountered. The constituent expression of this command is then evaluated in the mode determined by the prefix \textit{value of} or \textit{reference of}, and the results transformed if necessary to the appropriate types as defined in the previous paragraph. These results form the value of the block-expression and the execution of the block is then terminated.

Commands in a block-expression may include assignments which update L-values which are non-local to the block (i.e. which are not lost when the execution of the block is terminated). In such cases the block-expression is said to have SIDE EFFECTS, and particular care must be taken to ensure that the results of the program are not dependent on the order of evaluations or the rules given in Section 4.2 may imply that the results are unpredictable.
4.5 Expression-Lists

An expression may generally produce a sequence of results; where there are two or more of these, the syntactic class expression-list enables the members to be written explicitly separated by commas. This form is naturally only meaningful in contexts where several values are meaningful: for example, in assignmentCommands where several simultaneous assignments are being made (see Section 9.2), and in actual parameter lists where several values are being handed over as the operands of a function or routine call (see Section 10.1).

Note that whenever an expression produces a sequence of two or more values, these values are never considered together as being a single value; and thus such an expression does not have a single type, but a sequence of types.

Several or all of the members of an expression-list may be bracketed: for example, in order to delimit the scope of a where-clause. Otherwise, bracketing in an expression-list has no significance.
4.6 Where-Clauses

The general form of an expression qualified by a where-clause is

\[ E, \text{where} \ D, \]

where the syntax rules permit \( E \) to be any expression (possibly qualified by a where-clause itself) and \( D \) to be an in-definition (see Section 8.1). However, since commands and definitions may also be qualified by where-clauses, it is usually necessary for the above form to be enclosed in parentheses. For this reason, the syntactic rules for certain commands and declarations use the forms expression-list or basic-expression to indicate that, if the expression only is to be qualified by a where-clause, they must be enclosed in parentheses.

The semantics of an expression qualified by a where-clause are conveniently defined by the following equivalent form

\[ \text{ref of } \& \text{let } D; \text{ result is } E, \& \]

(See Sections 4.4 and 8.3).

The rules governing the scope of where-clauses may be changed to make them simpler and more logically coherent.
5  PREFIXED OPERATORS AND EXPRESSIONS

5.1 Monadic Operators
   5.1.1 Syntax
   5.1.2 Semantics

5.2 Prefixed-Operators
   5.2.1 Syntax
   5.2.2 Semantics

5.3 Prefixed-Expressions
   5.3.1 Syntax
   5.3.2 Semantics
   5.3.3 Array References
5.1 **Monadic Operators**

5.1.1 **Syntax**

\[
\langle \text{individual} \rangle ::= \langle \text{constant} \rangle | \langle \text{name} \rangle | \langle \text{prefixed expression} \rangle | \\
\sim \langle \text{individual} \rangle | \langle \text{block expression} \rangle | \text{rhs} | \\
\text{pos} | \text{neg} | (\langle \text{expression} \rangle)
\]

5.1.2 **Semantics**

(a) The operator \( \sim \)

This is written immediately before its operand, and is less binding than any prefixed-operator, but more binding than any infixed-operator (see Section 6.1). It takes an R-value as an operand (which must be of the type **logical**, **longlogical** or **Boolean**) and produces an R-value of the same type as its result.

If the operand is of type **logical** or **longlogical**, the nth bit of the result is determined by the nth bit of the operand according to the following table:

<table>
<thead>
<tr>
<th>Operand</th>
<th>0</th>
<th>1</th>
<th>( \sim )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>1</td>
<td>0</td>
<td>( \sim )</td>
</tr>
</tbody>
</table>

If the operand is of type **Boolean** the result is defined by the following table.

<table>
<thead>
<tr>
<th>Operand</th>
<th>false</th>
<th>true</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>

(b) Operators + -

+ and - may be used as monadic operators in publication CPL. However, the transformation to canonical CPL replaces them by the symbols pos and neg respectively. In the precedence and grouping rules for determining infixed-operators (see Section 6.1), these symbols are formally treated as equivalent to the symbol sequences (+1) and (-1) respectively. Thus the meaning of prefixed + and - is defined to be the same as that of multiplication by the system constants (+1) and (-1) which have the values 0+1 and 0-1 respectively.
5.2 Prefixed-Operators

5.2.1 Syntax

<pre>prefixed operator ::= <name> | rhs | prefixed expression | (<expression>)</pre>

5.2.2 Semantics

A prefixed-operator can be a name, a general expression enclosed in parentheses, or a prefixed-expression. If the prefixed-operator is a name, it must be either a BASIC FUNCTION (see Section 10.1,2) or a PROGRAMMER'S FUNCTION (see Section 10.3) or an ARRAY (see Section 11). If the prefixed-operator is an expression, it must be possible to evaluate it in R-mode to produce a function or array.
5.3 Prefix-Expressions

5.3.1 Syntax

<prefixed expression> ::= <prefixed operator>[[<expression>,...]]

A prefixed-expression consists of a prefixed-operator followed by its ARGUMENT LIST which is enclosed in SQUARE brackets. The brackets must be present even if the argument list is empty (i.e., if the operator requires no operands).

5.3.2 Semantics

A prefixed-expression is used to indicate the APPLICATION of a prefixed-operator to its operands. (Note that the EVALUATION of an operator and its APPLICATION are two completely distinct processes.) The evaluation of a prefixed-expression in either L-mode or R-mode is performed in two stages:

(a) The prefixed-operator is evaluated in R-mode to produce a function or array, and the operand (which is the expression enclosed in brackets following it) is evaluated in the appropriate mode to form the argument list. These two evaluations may be performed in any order.

(b) The resulting function or array is applied to its argument in the manner described in Section 10.1 to produce the value of the prefixed-expression. The application of a function to its arguments is known as a FUNCTION CALL.

In general, the operand is evaluated in L-mode and the resulting argument list of L-values, without changes of type, is used in the function call. However, in the special case where the written operator is the name of a function data item with the attribute constant, and whose formal parameter types and modes are explicitly (or by default) stated in the definition, the components of the argument list in a function call are further transformed (if possible) so that they match the corresponding formal parameters in mode and type. (Note that this special case will probably be the most common form of function call.)

A function may only be called with an argument list with the correct number of arguments. (Some basic functions may be called with any number of arguments - see Appendix 3. No programmer's functions can have this property.)
The type of a prefixed-expression is determined by the type of its prefixed-operator (see Section 10.2.2.).

5.3.3 Array References

An array-type data item may be used as a prefix-operator to refer to an element of an array. In this form, the operator takes a single operand of type index in R-mode (called a subscript), which is used to select the required element according to the definition of the array (see Section 11). The natural mode of evaluation of an array reference is L-mode, giving the L-value of an element of the array. The result could itself be an array, which may be applied to a further subscript to obtain a sub-element, and so on.

For example, if A represents a three dimensional array of real elements (i.e., of type real 3 array), and i, j, k represent index values

\[ A[i] \] gives an L-value of type real 2 array
\[ A[i][j] \] gives an L-value of type real 1 array
\[ A[i][j][k] \] gives an L-value of type real

An alternative syntactic form permits an array reference to be written with two or more subscripts to specify directly sub-elements of multidimensional arrays. For example:

\[ A[i,j] \] is equivalent to \[ A[i][j] \]
\[ A[i,j,k] \] is equivalent to \[ A[i][j][k] \]

The result of an array reference is only defined if the value of the subscript lies within the bounds of the array.
6 **INFIXED OPERATORS AND EXPRESSIONS**

6.1 Syntax and Grouping  
6.1.1 Syntax  
6.1.2 General  
6.1.3 Juxtaposition, Pos and Neg  
6.1.4 Grouping  

6.2 Numerical Operators  
6.2.1 Types  
6.2.2 Semantics  

6.3 Logical Operators  

6.4 Relations  

6.5 String Operators  

6.6 Polymorphism and Type Matching
6.1 Syntax and Grouping

6.1.1 Syntax

\[ \text{<relation> ::= muchlessthan | lessthan | < | = | ≠ | > | greaterthan | muchgreaterthan} \]

\[ \text{<infixed operator> ::= + | - | \times | / | \uparrow | = | ≠ | \land | \lor} \]

\[ \leftrightarrow \text{<relation> \bullet \text{<infixed operator> \bullet \text{<infixed expression>}}} \]

6.1.2 General

Infixed-operators require two operands; they may require operands of a particular type or be polymorphic (see Section 6.2 and 6.6). With the possible exception of the operators = and ≠, their operands are evaluated in R-mode and the result they produce is always an R-value.

In the written form of expressions, an infixed-operator is placed between its operands and together these may form an operand of a further operator. Thus, an expression may appear as a sequence of alternate operands (i.e. items of the syntactic class Individual) and infixed-operators; and therefore rules exist to determine the grouping of operators with operands in any such sequence. These rules may be overridden explicitly by enclosure of an operand in parentheses.

6.1.3 Juxtaposition, Pos and Neg

The symbols pos and neg rank as members of the syntactic class Individual. Semantically, they are treated as constant data items with the values +1 and -1 respectively, their type being that numerical type with the lowest precision (i.e. normally index).

The juxtaposition of two individuals in an infix-expression is taken as implying multiplication and the operator is therefore inserted before any further analysis takes place.

This may need some modification
6.1.4 Grouping

Operator \( \uparrow \times / - + \leftrightarrow \langle \text{relation} \rangle \wedge \vee = \neq \)

| PL | 10 | 9 | 9 | 6 | 6 | 5 | 4 | 3 | 2 | 1 |
| PR | 8  | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Table of Precedences

Grouping rules are determined by assigning two PRECEDENCES to each infixed-operator (a left precedence PL and a right precedence PR), and by specifying a procedure of analysis on an operand-operator sequence of the general form

\[ A_0 x_1 A_1 x_2 \ldots x_n A_n \]

where \( A_0, A_1, \ldots A_n \) are individuals and \( x_1, x_2, \ldots x_n \) are infixed-operators. The operator-operand sequence is preceded by a dummy operator \( x_0 \) (for which PR = 0) and followed by a dummy operator \( x_{n+1} \) (for which PL = 0); thus:

\[ x_0 A_0 x_1 A_1 x_2 \ldots x_n A_n x_{n+1} \]

The grouping procedure is the following. The sequence is searched for any subsequence \( A_i x_{i+1} \ldots x_j A_j \) such that

\[ 1 \leq i < j \]

\[ \text{PR}[x_i] < \text{PL}[x_{i+1}] \]

\[ \text{PR}[x_j] > \text{PL}[x_{j+1}] \]

and, for all \( s = i+1, \ldots j-1 \)

\[ \text{PR}[x_s] = \text{PL}[x_{s+1}] \neq 0 \]

A subsequence of this form is termed a PRIME PHRASE. Any prime phrase found is enclosed in parentheses and thus becomes an individual; the operator-operand sequence is then renumbered to take account of this. The process is repeated until no further prime phrases can be found.

This procedure will not group an infixed-expression completely into pairs of operands separated by an operator. In particular, the operators which are associative (viz \( \times + \wedge \vee = \) and \( \neq \)) may occur in groups with any number of members. These may, in the cases of \( \times \) and \( + \), be followed by a single occurrence of \( / \) or \( - \) respectively. Thus an infixed-expression such as:

\[ a + b + c - d \]

\[ p \times q / r \]

\[ x = y \neq z \]
are treated as individuals and not grouped any further. This, however, is in accordance with their normal mathematical meaning which implies that the ordering (or grouping) of the operations inside such a group has no effect on its value.

The relations are also combined into a single group. A COMPOUND RELATION, as such a group is called, has the Boolean value True if and only if each of its component relations has the value True. Thus the compound relation

$E_1 R_2 E_2 R_3 E_3 \cdots E_{n-1} R_n E_n$

has the value True if and only if all the relations

$E_1 R_2 E_2$
$E_2 R_3 E_3$
$\cdots$
$E_{n-1} R_n E_n$

has the value True.
6.2 Numerical Operators

6.2.1 Types

Infixed-operators are in general POLYMORPHIC; that is they may take operands of various types, and may produce results whose type depends upon that of the operands. However, in all cases, the operator takes two operands of the same type. (Two operands of differing types are permitted in certain circumstances, but these are always converted to a single common type before application of the operator: See Section 6.6.)

In this and following sections, a table is given for each operator (or group of operators) specifying the type of the result as a function of the type of the operands.

<table>
<thead>
<tr>
<th>Operators + - x</th>
<th>Operand Type</th>
<th>Result Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>index</td>
<td>index</td>
<td></td>
</tr>
<tr>
<td>integer</td>
<td>integer</td>
<td></td>
</tr>
<tr>
<td>real</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>double</td>
<td>double</td>
<td></td>
</tr>
<tr>
<td>complex</td>
<td>complex</td>
<td></td>
</tr>
<tr>
<td>double complex</td>
<td>double complex</td>
<td></td>
</tr>
<tr>
<td>general</td>
<td>general</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operators / ↑</th>
<th>Operand Type</th>
<th>Result Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>real</td>
<td>real</td>
<td></td>
</tr>
<tr>
<td>double</td>
<td>double</td>
<td></td>
</tr>
<tr>
<td>complex</td>
<td>complex</td>
<td></td>
</tr>
<tr>
<td>double complex</td>
<td>double complex</td>
<td></td>
</tr>
<tr>
<td>general</td>
<td>general</td>
<td></td>
</tr>
</tbody>
</table>
6.2.2 Semantics

The meaning of the numerical infixed-operators is intended to be the same as their normal mathematical meaning. In many cases, however, the correspondence is only approximate due to the presence of rounding and similar errors. Such divergences from the mathematical ideal are implementation dependent.

Infixed division (/) produces a minimum type of real. There is thus no infixed-operator producing the integer type of division with a remainder. These results can be obtained by using the basic functions Quot[x,y] and Rem[x,y] (see Appendix 3). The division symbol \( \div \) is not a part of CPL.

Infixed exponentiation (\( ^{} \)) is somewhat irregular. Like division its minimum type is real, but its result type is required to be the same as its operand type and for operands of types real or double this imposes a restriction on the values of the operands.

More precisely, the value of the expression \( E_1 E_2 \) is one of the values of \( \exp(E_2 \log E_1) \) with the following restrictions:

1) If \( E_1 \) and \( E_2 \) are of type real or double, the expression is undefined unless at least one of the values of \( \exp(E_2 \log E_1) \) is not complex.

2) If \( E_1 \) and \( E_2 \) are of type real or double and \( \exp(E_2 \log E_1) \) has two real values, the value of \( E_1 E_2 \) is the positive one of these.

3) If \( E_1 \) and \( E_2 \) are of type complex or double complex and \( \exp(E_2 \log E_1) \) has more than one value, the choice between them is implementation dependent.
6.3 **Logical Operators**

<table>
<thead>
<tr>
<th>Operators</th>
<th>( \land )</th>
<th>( \lor )</th>
<th>( = )</th>
<th>( \neq )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operand Type</td>
<td>logical</td>
<td>logical</td>
<td>Boolean</td>
<td>general</td>
</tr>
<tr>
<td>Result Type</td>
<td>logical</td>
<td>long logical</td>
<td>Boolean</td>
<td>general</td>
</tr>
</tbody>
</table>

Where these operators produce *logical* or *long logical* results, they are defined to be bit-by-bit manipulations on the individual bits of the operands. Thus, the nth bit of the result is determined by the nth bit of each operand according to the following table.

1st operand | 0 | 0 | 1 | 1 |
2nd operand | 0 | 1 | 0 | 1 |

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \land )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \lor )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( = )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \neq )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

When the operators produce *Boolean* results, their values may be determined from the same table by replacing 0 by *false* and 1 by *true*. 
6.4 Relations

Operators = ≠

Operand Type         Result Type
    any          Boolean

If the operands are one of the numerical types, then

\[ E_1 = E_2 \]

is true if \((E_1 - E_2)\) is a representation of zero. Thus, as with subtraction, equality of numerical values is implementation dependent.

For all other types, \(E_1 = E_2\) is true if the operands yield identical R-values (but see Section 10.2.4).

The expression

\[ E_1 ≠ E_2 \]

is always equivalent to

\[ \sim (E_1 = E_2) \]

Note that, if the operands are of different types, transfer functions may be invoked under the rules of Section 6.6. For equality between operands of type function or routine see Section 10.2.4, and between operands of type array see Section 11.1.2.

Operators < ≤ ≥ >

Operand Type         Result Type
    Index          Boolean
    Integer        Boolean
    real           Boolean
    double         Boolean
    string         Boolean

If the operands \(E_1\), \(E_2\) are of one of the permitted numerical types (which are all non-complex) the value of the relation is determined from the value of \(E_1 - E_2\). Transfer functions may be invoked (see Section 6.6) to make this
possible.

If the operands are of type `string`, comparison is
affected by repeatedly comparing the numerical equivalent
(see Section 3.3.5) of successive characters of each string
value (beginning with the first or most significant
character) until the required attribute is established. If
the characters of one of the operands are exhausted before
the other in this process, it is assumed to continue with
sufficient dummy characters whose numerical equivalents are
less than any other character.

### Operators `<<` `>>`

<table>
<thead>
<tr>
<th>Operand Type</th>
<th>Result Type</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>index</code></td>
<td><code>Boolean</code></td>
</tr>
<tr>
<td><code>integer</code></td>
<td><code>Boolean</code></td>
</tr>
<tr>
<td><code>real</code></td>
<td><code>Boolean</code></td>
</tr>
<tr>
<td><code>double</code></td>
<td><code>Boolean</code></td>
</tr>
</tbody>
</table>

These operators formally give the result `true` if the
value of one is negligible compared with the value of the
other. This normally means that

\[
E_1 \ll E_2 \text{ is equivalent to } (E_1 + E_2) = E_2 \\
E_1 \gg E_2 \text{ is equivalent to } (E_1 + E_2) = E_1
\]

The precise definition however is implementation dependent,
and in some floating point implementations of the types `real`
and `double` the decision may be made in the basis of the
value of the exponent alone, so that \(E_1 \ll E_2\) may sometimes
be stronger than \(E_1 + E_2 = E_2\) and sometimes be weaker.

If \(E_1\) is of type `index` or `integer` and does not have the
value zero, \(0 \ll E_1\) is always `true`. The value of \(0 \gg 0\) is
implementation dependent but should normally be `false`. 
6.5 **String Operators**

<table>
<thead>
<tr>
<th>Operator</th>
<th>Operand Type</th>
<th>Result Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>string</td>
<td>string</td>
</tr>
</tbody>
</table>

This operator concatenates two strings. That is, the result is a string consisting of the characters of the first operand, followed by the characters of the second operand.

The introduction of the `type character my involve alterations to the section.`
6.6 Polymorphism and Type Matching

The infixed-operators described in Sections 6.2, 6.3 and 6.4 are all polymorphic. The general rules for choosing the correct version of a polymorphic operator and inserting any necessary transfer functions are given in Sections 3.3.3 and 3.3.4. The choice for all these operators is made using Rule A of Section 3.3.4.
7 DEFINITIONS

7.1 Syntax

7.2 Modes of Definition
   7.2.1 Definition by Type
   7.2.2 Definition by Value
   7.2.3 Definition by Reference

7.3 Constant and Variable Definitions

7.4 Definitions and Types
   7.4.1 Preferred Type
   7.4.2 Type Definitions
   7.4.3 Simple Initialized Definitions
7.1 Syntax

<data type> ::= real | Integer | complex | double | logical | index | longlogical | doublecomplex | type | Boolean | label | routine | string | general

<array type> ::= <single type>, <number> array | <single type>, <vector | matrix>

<function type> ::= <single type>, function

<single type> ::= <data type> | <array type> | <function type> | (<single type list>)

<single type list> ::= <single type>, <single type>,...

<name list> ::= <name>, <name>,...

<formal parameter> ::= <single type> <value | reference> <name> | <value | reference>, <single type>, <name>

<formal parameter list> ::= <formal parameter>, <formal parameter>,...

<type definition> ::= <name list> <be | allbe> <single type list>

<simple definition> ::= <variable | constant> <name list> <expression list>

<LH function body> ::= <fix <definition>>, <load <block> update <block>

<function definition> ::= <recursive>, <variable | constant>, <name> [<formal parameter list], <expression list> | <recursive>, <variable | constant>, <expression list> | <recursive>, <variable | constant>, <function> <name> [<formal parameter list], be <LH function body>]

<routine definition> ::= <recursive>, <variable | constant>, <fixed | free>, <routine> <name> [<formal parameter list], be <block>

<basic definition> ::= <type definition> | <simple definition> | <function definition> | <routine definition> | <recursive>
7.2 Modes of Definition

One of the functions of a definition is to introduce a name which refers to a data item, and which is then identified with all other occurrences of the same name in the scope of the definition. The scope rules are given in Section 8. Other characteristics depend on whether the definition is by value or reference.

7.2.1 Definition by Type

Type-definitions have the defining operator be or allbe. They create a new L-value (i.e. one which is disjoint from all other existing at the moment of definition) and associate this with the defined name. The R-value associated with this new L-value is undefined.

7.2.2 Definition by Value

A definition by value creates a new L-value and associates this with the defined name; it also associates an initial R-value with the L-value. Definitions by the defining operators =, all = and the special forms of function and routine definitions all define by value.

7.2.3 Definition by Reference

A definition by reference is primarily intended to associate an already existing L-value with the name defined. The L-value is the one obtained by evaluating the defining expression in L-mode. In the exceptional case where the natural mode of evaluation of the defining expression is R-mode, a new L-value of the appropriate type is created, its R-value is the result obtained by evaluating the defining expression in R-mode. This L-value is then associated with the defined name.
7.3 **Constant and Variable Definitions**

An L-value in CPL has either the property `constant` or `variable`, depending solely on how the L-value was created. If an L-value has the property `constant`, it means that its R-value cannot be changed by assignment. It is meaningful to include the word `variable` or `constant` only in definitions by value, if it is omitted from a definition, a constancy attribute is assumed. A function or routine definition is assumed to have the attribute `constant` unless explicitly specified as `variable`, whereas a type or simple definition is taken as `variable`, unless otherwise specified.
7.4 Definitions and Types

Data items of any type may be defined; however the actual type in most cases need not be explicitly stated. The rules for determining the type depend on the kind of definition, and on the current PREFERRED TYPE.

7.4.1 Preferred Type

There are many occasions when it is necessary to determine the type of a data item in CPL. In most cases this can be deduced from known or ascertainable types of its component parts, but there are a few situations where this is not possible. The most important of these are: in the formal parameter lists of function and routine definitions (see Section 10), and in written numerical constants. In these situations it is always possible to specify the type required explicitly.

It is possible for the programmer to select a preferred type which will be used in certain circumstances, where the type of a data item is otherwise unknown.

Initially the preferred type is real; it will be altered only if the programmer writes a directive in the form

prefer <single type>

where the <single type> specifies the new preferred type. This directive may be written in the declaration sequence of a block; its scope consists of any succeeding declarations, together with the command sequence of the block.

If the programmer has specified the type of an array, matrix, vector or function without specifying the single-type which determines the type of its components or result, this is assumed to be the preferred type. This does not apply to functions defined by one of the special forms of definition described in Section 10.3 whose result types are discussed in Section 10.3.6.

7.4.2 Type Definitions

(a) The defining operator be

The syntax of the simplest form of type definition is

<name list> be <single type list>
It is only meaningful if the name-list and single-type-list have the same number of members, in which case the names on the left of the operator be are associated with data items whose types are the corresponding single-types of the single-type-list. Initial values for the data items are not defined.

(b) The defining operator \textbf{all be}
This operator may be used when it is necessary to define by type a number of data items all of the same type. For example, the following two definitions have the same meaning.

\[
x, y, z \text{ be real, real, real}
\]
\[
x, y, z \text{ all be real}
\]

7.4.3 \textbf{Simple Initialized Definitions}

(a) The defining operator \textbf{=}
The type of each of the data items on the left is the type of the corresponding expression on the right. There must be the same number of defining expressions on the right as names in the name-list on the left. If any of the defining expressions is a single numerical constant and the preferred type is numerical then the type of this constant is determined by the rules given in Section 3.2.2. This form of definition defines by value (see Section 7.2.3).

(b) The defining operator \textbf{\ensuremath{\wedge}}
As in (a) the type of each of the data items on the left is the type of the corresponding expression on the right, and there must be the same number of defining expressions as names defined. For each name defined the corresponding defining expression is evaluated in L-mode, and the L-value obtained is associated with the name. (See Section 7.2.3.)

(c) The defining operators \textbf{all =} and \textbf{all \ensuremath{\wedge}}
The effects of these operators are best described in terms of the operators described above.
For example, the following two definitions are synonymous:

\[
a, b, c \text{ all = } E_i
\]
\[
a, b, c = (x, x, x \text{ where } x = E_i)
\]

Similarly the following two are also synonymous:

\[
a, b, c \text{ all } \ensuremath{\wedge} E_i
\]
\[
a, b, c \ensuremath{\wedge} (x, x, x \text{ where } x \ensuremath{\wedge} E_i)
\]
8 DEFINITION STRUCTURE AND SCOPE RULES

8.1 Syntax

8.2 Scope and Extent
  8.2.1 Scope
  8.2.2 Extent

8.3 Scope Rules for Definitions
  8.3.1 Recursive
  8.3.2 Composite Definitions
  8.3.3 And
  8.3.4 In
  8.3.5 Where
  8.3.6 Let

8.4 Other Scope Rules
8.1 Syntax

<and definition> ::= <basic definition>
                < and <basic definition>>>.

<in definition> ::= <and definition> |
                 <and definition> in <in definition>

<definition> ::= <in definition> |
               <definition> where <in definition>

<declaration> ::= let <definition>

The scope of where may be changed.
8.2 Scope and Extent

8.2.1 Scope

The SCOPE of a definition is a syntactic concept; it is that area of the written program in which the data items defined in definitions may be referred to using the names with which the definitions associate them.

If a definition (the 'inner' definition) occurs within the scope of another definition of the same name (the 'outer' definition), the inner definition supersedes the outer one. The outer definition is said to be 'shielded' from the scope of the inner one, and to have a 'hole' in its scope.

8.2.2 Extent

The EXTENT of a data item is a dynamic concept; it is that part of the dynamic execution of a program through which a named data item maintains a continuous existence.

In CPL the extent of a named data item is controlled by the scope of its definition. It continues as long as the command currently being executed lies within the scope (including any holes there may be in it); it terminates as soon as the current command lies outside the scope of the definition.

For this purpose the whole of the execution of a routine call is considered to lie in the extent containing the call, irrespective of the scope which contains the body of the routine.
8.3 Scope Rules for Definitions

The scope of a definition is controlled by its position in a program, and by use of the words recursive, and, in and where.

8.3.1 Recursive

The scope of a definition does not normally include its own definiens (i.e., the right hand side of its own definition). However, by preceeding the definition by the word recursive, it can be made to do so. This may only be used if all the data items defined by the definition are either functions or routines. The effect of recursive on other definitions is undefined.

8.3.2 Composite Definitions

Composite definitions made up of definitions joined by and, in or where together with uses of recursive may be formed into a single basic definition by enclosing them in section brackets. The resulting definition may also be qualified by preceeding it by the word recursive provided it satisfies the conditions of Section 8.3.1.

8.3.3 And

This word is used to combine two or more definitions into one. Unless this combined definition is recursive, none of the component definitions is in the scope of any other component definition. The order of activation of the definitions is undefined; it is intended that they be considered as activated in parallel. Thus, for example, the effect of the definitions.

\[ N_1 = E_1 \]
\[ \text{and } N_2 = E_2 \]
\[ \ldots \ldots \]
\[ \text{and } N_r = E_r \]

is the same as the effect of the definition

\[ N_1, N_2, \ldots, N_r = E_1, E_2, \ldots, E_r \]
The effect is undefined unless the names $N_1, N_2, \ldots, N_r$ are all different.

8.3.4 In

When this word separates a number of definitions there is an implied association rule to the right. For example, the definition

$$D_1 \text{ in } D_2 \text{ in } D_3$$

is synonymous with

$$\text{ D}_1 \text{ in } \text{ D}_2 \text{ in } \text{ D}_3 \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \text{
The word where may also be used to introduce a where-clause qualifying an expression or command. The general rule is that a where-clause qualifies the longest command, definition or expression (in that order of preference) immediately preceding it; the scope of its component definition is the qualified command, definition or expression. As commands and definitions frequently terminate in expressions, it is generally necessary to enclose expressions qualified by where-clauses in parentheses. (See Sections 4.6 and 9.8.)

8.3.6 Let

A declaration is a definition preceded by the word let. A declaration or a sequence of declarations may only occur at the head of a block. The scope of a declaration is the set of immediately following declarations and the command sequence of the block. If the definition in the declaration is recursive then its scope includes its definiens as well.
8.4 Other Scope Rules

The concept of scope also applies in situations where names are introduced other than by definition. There are two cases, as follows.

(a) Formal Parameters
The scope of a formal parameter in a function or routine definition is the defining expression or command. (see Section 10.)

(b) Labels
The scope of labels declared by colon as in the command

\[ L : a := bx+c \]

is the smallest enclosing routine body or block expression (see Section 9.8).
9 Commands

9.1 Syntax

9.2 Assignment-Commands
   9.2.1 Syntax
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9.3 Transfer-Commands and Labels
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9.5 Other Simple-Commands
   \checkmark 9.5.1 Syntax
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9.6 Conditional-Commands
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   9.7.1 Syntax
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\checkmark 9.8 Blocks
   \checkmark 9.8.1 Syntax
   \checkmark 9.8.2 Notes
   \checkmark 9.8.3 Declarations
   \checkmark 9.8.4 Command-Sequences
   \checkmark 9.8.5 Leaving Blocks
9.1 Syntax

\[
\langle \text{assignment command} \rangle ::= \langle \text{expression} \rangle ::= \langle \text{expression list} \rangle | \\
\langle \text{expression} \rangle \text{ all} ::= \langle \text{basic expression} \rangle \\
\langle \text{transfer command} \rangle ::= \text{goto} \langle \text{basic expression} \rangle \\
\langle \text{routine command} \rangle ::= \langle \text{name} \rangle | \langle \text{prefixed expression} \rangle \\
\langle \text{simple command} \rangle ::= \langle \text{assignment command} \rangle | \langle \text{transfer command} \rangle | \\
\langle \text{routine command} \rangle | \langle \text{resultis} \rangle \langle \text{expression} \rangle | \\
\text{break} | \text{return} | \langle \text{block} \rangle \\
\langle \text{if command} \rangle ::= \langle \text{if} | \text{unless} \rangle \langle \text{expression} \rangle \text{ do} \langle \text{basic command} \rangle \\
\langle \text{test command} \rangle ::= \text{test} \langle \text{expression} \rangle \text{ do} \langle \text{command} \rangle \text{ or} \\
\langle \text{basic command} \rangle \\
\langle \text{conditional command} \rangle ::= \langle \text{if command} \rangle | \langle \text{test command} \rangle \\
\langle \text{while command} \rangle ::= \langle \text{while} | \text{until} \rangle \langle \text{expression} \rangle \text{ do} \\
\langle \text{basic command} \rangle \\
\langle \text{repeat command} \rangle ::= \langle \text{command} \rangle \langle \text{repeatwhile} | \text{repeatuntill} \rangle \\
\langle \text{command} \rangle \text{ repeat} \\
\langle \text{for element} \rangle ::= \langle \text{basic expression} \rangle | \\
\text{step} \langle \text{basic expression} \rangle, \langle \text{basic expression} \rangle, \\
\langle \text{basic expression} \rangle | \\
\langle \text{basic expression} \rangle \text{ to} \langle \text{basic expression} \rangle | \\
\langle \text{basic expression} \rangle, \langle \text{basic expression} \rangle \\
\text{dotstring} \langle \text{basic expression} \rangle \\
\langle \text{for list} \rangle ::= \langle \text{for element} \rangle | \langle \text{for element} \rangle \\
\langle \text{for list} \rangle \langle \text{where} \rangle \langle \text{in definition} \rangle \\
\langle \text{for command} \rangle ::= \langle \text{forexternal} \rangle \langle \text{individual} \rangle \langle \text{name} \rangle = \langle \text{for list} \rangle \text{ do} \langle \text{basic command} \rangle | \\
\langle \text{for command} \rangle \\
\langle \text{cycle command} \rangle ::= \langle \text{while command} \rangle | \langle \text{repeat command} \rangle | \\
\langle \text{for command} \rangle \\
\langle \text{basic command} \rangle ::= \langle \text{name} \rangle ::= \langle \text{conditional command} \rangle | \\
\langle \text{simple command} \rangle | \langle \text{cycle command} \rangle \\
\langle \text{command} \rangle ::= \langle \text{basic command} \rangle | \langle \text{command} \rangle \langle \text{where} \rangle \langle \text{in definition} \rangle \\
\langle \text{note} \rangle ::= \langle \text{prefer} \rangle \langle \text{single type} \rangle | \langle \text{sic} \rangle | \langle \text{note} \rangle \langle \text{string constant} \rangle \\
\langle \text{block} \rangle ::= \langle \text{note} \rangle, \langle \text{declaration} \rangle | \langle \text{declaration} \rangle, \langle \text{command} \rangle | \\
\langle \text{command} \rangle | \\
\langle \text{command} \rangle
9.2 Assignment-Commands

9.2.1 Syntax

\[
\text{<assignment command>} ::= \text{<expression>} ::= \text{<expression list>}|
\]
\[
\text{<expression>} \text{ all} ::= \text{<basic expression>}
\]

9.2.2 Semantics

(a) The effect of the command

\[ E_1 := E_2 \]

is as follows. The expression \( E_1 \) is evaluated in L-mode to produce a sequence of L-values of any type, say \( L_1, L_2, \ldots, L_m \); the expression \( E_2 \) is also evaluated in R-mode to produce a sequence of R-values of any type, say \( R_1, R_2, \ldots, R_n \); these evaluations may be performed in any order, but must be complete before the next phase starts. In order for the command to be well-formed, it is necessary that \( m = n \). Each member of the sequence \( L_1, L_2, \ldots, L_m \) is then UPDATED by the corresponding member of the sequence \( R_1, R_2, \ldots, R_n \).

To update the L-value \( L_k \) with the R-value \( R_k \), \( R_k \) is first transformed to be of the same type as \( L_k \) if the appropriate transfer function exists (if it does not, the effect of the command is undefined). Let the transformed R-value be \( R'_k \). The R-value associated with \( L_k \) is now altered to be \( R'_k \) so that future evaluations of \( E_1 \) in R-mode will produce the R-value \( R'_k \) (unless modified by further assignment-commands).

Note that the L-value \( L_k \) is not altered by updating it, and that the R-value of any expression which has the L-value \( L_k \) is altered by updating \( L_k \). If two L-values are not disjoint - i.e. if they share a part or the whole of their associated R-values - then updating either will effect the R-value associated with both at least as far as their shared part is concerned.

The order in which the sequence \( L_1, L_2, \ldots, L_m \) is updated is unspecified (cf. Section 4.2). This means that in general the L-values in the sequence \( L_1, L_2, \ldots, L_m \) should be disjoint or the effect of the assignment-command may be undefined.

(b) The effect of the command

\[ \text{E}_1 \text{ all} := \text{E}_2 \]
is as follows. The expression $E_1$ and $E_2$ are evaluated as described above to yield the sequences $L_1, L_2, \ldots, L_m$ and $R_1, R_2, \ldots, R_m$. In this case the sequence of $R$-values must have only one member (i.e. $n=1$).

The command is completed by updating every member of the sequence $L_1, L_2, \ldots, L_m$ with the $R$-value $R_1$.

9.2.3 Updating

If $L_1$ is an $L$-value and $R_1$ is an $R$-value of the same type, there is an operation known as updating $L_1$ with $R_1$. This operation depends on the way in which $L_1$ originally arose.

This section is incomplete.
9.3 Transfer-Commands and Labels

9.3.1 Syntax

<transfer command> ::= goto <basic expression>

<basic command> ::= <<name> ::> o <<conditional command> | 
                         <simple command> | <cycle command>

9.3.2 Semantics

In a transfer-command, the basic symbol goto is followed by a basic-expression which is to be evaluated to produce an R-value of type label.

The process of executing a command is known as an "activation" of that command. At any moment in the activation of a command, a number of further activations may be current, since one activation may call for a succession of further activations to be completed before it can be completed itself. Thus activation of a routine-command calls for an activation of the routine body which may call for activation of some sub-block of the body, and so on, each activation being "called for" by the most recent previous activation of the sequence which is not yet complete. If a function or routine is called recursively, the same command may have more than one activation current at the same moment.

An activation may be terminated either normally, as described elsewhere, or by a jump; a jump has an R-value of type label associated with it. This value specifies the destination of the jump, providing the following information:

(a) a point in some configuration of commands at which some activation is to be continued.

(b) a rule for determining which activation is to be continued at the point specified; this is in the form of a rule for determining whether or not an activation is "live" to the destination.

The effect of a transfer-command is then specified by the following rule, which is to be applied iteratively: activation of a transfer-command is immediately terminated by a jump whose destination is specified by the value of the expression associated with the command. If an activation calls for an activation which is terminated by a jump to a destination to which it is not itself live, then it also is terminated by the jump.
(For rules governing the termination of blocks and cycle-commands, see 9.8.5.)

This process is continued until an activation is found which is
live to the destination.

The effect of a jump is undefined if there is no current
activation to which it is live.

The following trivial example illustrates the way in
which ambiguities may arise when recursive routines and functions
are involved, and how this rule resolves them. If \( R \) is defined
by

\[
\text{let recursive routine } R \left[ \text{integer } n, \text{ label } L, M \right] \text{ be }
\]
\[
\begin{align*}
& \text{if } n > 0 \text{ do } R \left[ n-1, M, N \right] \quad \text{N : goto } L
\end{align*}
\]

the effect of executing \( R \left[ 10, \text{Even}, \text{Odd} \right] \), say, is to jump to
the destination "Even", whereas \( R \left[ 9, \text{Even}, \text{Odd} \right] \) jumps to the
destination "Odd". Note that each call of \( R \) ultimately has its
two label parameters specifying the same point \( N \) in two different
activations of \( R \).

Once the correct activation has been discovered, the
point at which it is to be resumed may still lie within blocks,
conditional-commands or cycle-commands which are not yet act-
ivated. (The rules of 9.3.3. will imply that these are the
only possibilities; in particular, note that the effect of
attempting to jump into an expression is undefined.) These
activations are achieved by successively "jumping into" the
commands, starting with the largest command which is not yet
activated and working inwards.

The effect of jumping into the command

\[
\text{if } E_1 \text{ then } C_1
\]

is identical with the effect of jumping into the result of
replacing this conditional command, until terminated, by \( C_1 \).

The effect of jumping into the command

\[
\text{for } N_1 = F_1 \text{ do } C_1
\]

is to execute the command as written, with the exception that
on the first cycle, if any, \( C_1 \) is entered by continuing the
jump into it, after setting the initial value of \( N_1 \).

On jumping into a block, any initialisations are per-
formed as if entering the block normally, before continuing
the jump further into the body of the block.
If, while evaluating the expressions occurring in a block or cycle command head for this purpose, a transfer-command is executed which terminates the block or cycle-command, this jump supersedes the former one, which is not pursued further.

The effect of attempting to jump from the head of a block or cycle-command into its body is implementation dependent.

The effects of jumping into other forms of compound-command may be deduced from the rules for expressing such commands in terms of commands of the above form.

The effect of a transfer-command which according to the above rules would terminate a call to the load or update part of a load-update pair, is implementation dependent, and may be undefined. (See Sec.10.3.5.)

9.3.3 Labels

An identifier of type label may be introduced in one of two different ways: by a definition using one of the basic symbols let, and, in and where, as specified in Sections 7 and 8; or by its occurrence as a "command label", in which case it occurs in the text followed by a colon, and preceding some command (or some further command label for a command).

A command label has the status of a definition of type label for the identifier which is used, in that a new data item is associated with that identifier whenever the scope of the command label is entered. The scope is that which a definition by let would have if placed at the head of the smallest routine body or block-expression body in which the command label occurs. The effect when two command label occurrences use the same identifier and have the same scope is undefined (see also Note 1.)

The data item associated with the command label identifier has an $R$-value which specifies a destination for jumps, providing the information for (a), (b) of 9.3.2 as follows:

(a) the point indicated is the point of the command label occurrence.

(b) the activation of the routine or block-expression body which is being initiated is live to the destination; an activation of a block, conditional-command or cycle-command will be live to the destination if its text contains the command label occurrence, and if called for by a command which is also live to the destination.

The data item associated with a command label is a constant, and may not be updated. But its $R$-value may be
assigned to variables of type \texttt{label} (declared at the head of a block, say).

Excepting possibly for an implementation dependent set of library labels, every \texttt{R} - value of type \texttt{label} originates as the value associated with a command label in the above way. For the sake of the \texttt{boolean} relations \texttt{=} \& \texttt{#} applied to labels, two such \texttt{R} - values are equal if they arise from (possibly different) labels for the same command at the same activation. Rules for equality between library labels are implementation dependent.

\textbf{NOTE}

1. For convenience in implementing labels, the interpretation is left implementation dependent in two situations of no great significance.

   (a) when, according to the above rules, a command label occurrence does not lie within its own scope.

   (e.g.

   \begin{verbatim}
   let routine R be \$\$ ; let L be real ; L : ... \$\$ ...
   \end{verbatim}

   (b) when the scope is a routine body, and the command label identifier is also used as a formal parameter for the routine.
9.4 Routine-Commands

9.4.1 Syntax

\[ \text{<routine command>} ::= \text{<name> | <prefixed expression>} \]

\[ \text{<prefixed expression>} ::= \text{<prefixed operator>}[\text{<expression>}] \]

9.4.2 Semantics

(a) If \( N_i \) is a name the command

\[ N_i \]

has the same effect as the command

\[ N_i[\] \]

(b) If a command is a prefixed-expression, the operator and operand are evaluated in the manner described in Section 5.3.2. The evaluation of the prefixed-operator, in this case, however, must produce a routine. The rules about evaluating the operand and transforming the resulting arguments to the appropriate mode and type are those given in Section 5.3.2.

The application of a routine to its arguments is a form of command; its effect is described in Section 10.1.
9.5.3 Break

The basic symbol break is a command which may only appear within the controlled command of cycle-command. It may not appear in the body of a routine definition or in the block following the basic symbol update unless the cycle-command immediately enclosing it textually does so also (see Section 10.3).

It has the same effect as a transfer-command to a label immediately following the smallest cycle-command enclosing it textually.

9.5.4 Result-is

The command

```
result is E
```

may appear in the block following one of the basic symbols value of, reference of or load. It may not appear in the body of a routine definition or in the block following the basic symbol update unless the basic symbol with which it is associated does so also (see Section 10.3).

The effect of the command is described in Sections 4.4 and 10.3.5.

These sections may need major revision.
9.6 Conditional-Commands

9.6.1 Syntax

\(<\text{if command}\> ::= \langle\text{if} \mid \text{unless} \rangle \langle\text{expression}\> \text{ do } \langle\text{basic command}\>

\(<\text{test command}\> ::= \text{test} \langle\text{expression}\> \text{ do } \langle\text{command}\> \text{ or } \langle\text{basic command}\>

9.6.2 If Commands

The command sequence

\[
\text{If } E_1 \text{, do } C_1 \\
C_2
\]

has the following effect:

The expression \(E_1\) is evaluated in R-mode to produce a result of type \text{Boolean}. If the value of \(E_1\) is \text{false}, the command \(C_1\) is skipped and the execution continues with the command \(C_2\); if the value of \(E_1\) is \text{true}, the execution continues with the command \(C_1\). If the R-value of \(E_1\) is not transformable to type \text{Boolean}, the effect is undefined.

The command

\[
\text{unless } E_1 \text{ do } C_1
\]

has the same effect as the command

\[
\text{if } \sim (E_1) \text{ do } C_1
\]

9.6.3 The Test-Command

The command

\[
\text{test } E_1 \text{ do } C_1 \text{ or } C_2
\]

has the same effect as the command

\[
\text{$\$ if } B_1 \text{, do } C_1 \\
\text{unless } B_1 \text{ do } C_2 \text{ $ where } B_1 = E_1
\]

provided the name \(B_1\) used for the value of the Boolean expression \(E_1\) is chosen to be distinct from all other names occurring in the command.
9.7 Cycle-Commands

9.7.1 Syntax

<while command> ::= <while | until > <expression> do <basic command>

<repeat command> ::= <command> <repeatwhile | repeatuntil > <basic expression> |
<command> repeat

<for element> ::= <basic expression> |
step <basic expression>, <basic expression>,
<basic expression> |
<basic expression> to <basic expression> |
<basic expression>, <basic expression>
dotstring <basic expression>

<for list> ::= <for element>,<, <for element> > |
<for list> where <in definition>

<for command> ::= for <name> = <for list> do <basic command> |
forexternal <individual> < = | := > <forlist> do <basic command>

<cycle command> ::= <while command> | <repeat command> |
<for command>

Either := or = may be used in either form of for-command

9.7.2 While-Commands

(a) The command
while E₁ do C₂
has the same effect as the command
L₁: if E₁ do $ C₁ go to L₁, $

(b) The command
until E₁ do C₁
has the same effect as the command
while ¬(E₁) do C₁
9.7.3 Repeat-Commands

(a) The command

\[ C, \text{ repeat} \]

has the same effect as the command

\[ \text{while true do } C, \]

(b) The command

\[ C, \text{ repeat while } E, \]

has the same effect as the command

\[ C, \text{ unless } E, \text{ do break } C, \text{ repeat} \]

(c) The command

\[ C, \text{ repeat until } E, \]

has the same effect as the command

\[ C, \text{ repeat while } \sim(E,) \]

9.7.4 For-Commands

(a) The commands

\[ \text{for external } l, := F, \text{ do } C, \text{ and } \]
\[ \text{for external } l, = F, \text{ do } C, \]

have the same effect as the command

\[ \$ \text{ let } N, = E, \]
\[ \text{for } N, = F, \text{ do } C, \text{ and } \]$ $N, := N, \$ \$

provided \(N,\) and \(N,\) are names which are distinct from any other names occurring in the command.

(b) The command

\[ \text{for } N, = F, \text{ do } C, \]
is interpreted as follows. The for-list $F_i$ is evaluated in R-mode as described in Section 9.7.5 to specify a sequence of R-values. The name $N_i$ is taken as the name of a new CONTROL variable whose type is the least upper bound of the types of the elements in this sequence and whose scope is the controlled command $C_i$.

If the sequence of R-values obtained by evaluating $F_i$ is void, the for-command has no effect. Otherwise, if the sequence of R-values obtained is $V_1, V_2, \ldots, V_n$ ($n \geq 1$) the effect of the for-command is the same as the effect of the command

$\$ $\$ let N_i = V_1; C_1$ $\$
$\$ $\$ let N_i = V_2; C_2$ $\$
$\$ $\$ \ldots \ldots \ldots \ldots \ldots$ $\$
$\$ $\$ let N_i = V_n; C_n$ $\$

Note that the values $V_1, V_2, \ldots, V_n$ are determined once and for all before the cycle is entered, so that they cannot be altered by assignments within the controlled command, and that the control variable $N_i$ is redefined with the next value of the sequence at the start of each cycle, so that assignments to $N_i$ are effective only until the end of the cycle in which they are made.

A jump out of the controlled command either by a transfer-command to a label external to it or by a break command, has the effect of jumping out of a block so that the control variable $N$ is lost.

A jump into the controlled command is treated like a jump into a block (see Section 9.8) and the evaluation of the for-list is treated as part of the declarations at the head of the block. More precisely, the effect of the commands

\begin{verbatim}
  go to L_1,
  ********
  for N_i = F_i do
  \$C_1; \ldots; L_1 : C_2; \ldots\$
\end{verbatim}

is the same as that of the commands

\begin{verbatim}
  go to L_2
  ********
  L_2 : $\$ let B_i = True
  for N_i = F_i do
  $\$ if B_i do $\$ B_i := False
  $\$ go to L_1 $\$
  \$C_1; \ldots; L_1 : C_2; \ldots\$
\end{verbatim}

provided the new names introduced are distinct from all others. Note that this equivalence, unlike many given elsewhere in this section, is only true dynamically. There is no simple textually equivalent sequence of commands as the interpretation of $L_i$ as a label in a transfer-command
depends on its context, i.e., on whether the transfer-command is inside or outside the for-statement which contains \( L \).

### 9.7.5 Evaluation of For-Lists

The sequence of values specified by a for-list is the sequence of \( R \)-values of its component for-elements taken in order from left to right. A for-element which is a basic-expression specifies a single value; the other forms specify an arithmetic progression (which may have no members) determined as follows:

(a) The for-element

\[
\text{step } E_1, E_2, E_3
\]

where \( E_1 \), \( E_2 \) and \( E_3 \) are numerical expressions specifies a sequence with \( n+1 \) values where \( n=\text{integer}[\text{Realpt}((E_3-E_1)/E_2)] \) if \( n \geq 0 \) and an empty sequence if \( n < 0 \). Here the function Realpt gives the real part of its argument for complex numbers, and the function Integer gives the nearest integer to its argument (see Appendix 3). If \( n \geq 0 \) the sequence of values specified is given by \( E_1+rE_2 \) for \( r = 0, 1, \ldots, n \). The type of each element of a sequence which is not void is the type of the expression \( E_1+E_2 \).

(b) The for-element

\[
E_1 \text{ to } E_2
\]
specifies the same sequence as the for-element

\[
\text{step } E_1, 1, E_2
\]

(c) The for-element

\[
E_1, E_2, \ldots, E_3
\]
specifies the same sequence as the for-element

\[
\text{step } E_1, E_2-E_1, E_3
\]

(d) If the for-list is qualified by a where-clause, the scope of the definition is the for-list only. Thus in the command

\[
\text{for external } E_i := F_i \text{ where } D_i \text{ do } C_i
\]

the scope of \( D_i \) includes all the elements of \( F_i \), but does not include either \( E_i \) or \( C_i \).
10 FUNCTIONS AND ROUTINES

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10.3 Function and Routine Definitions
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10.1 Introduction. Function and Routine Calls

10.1.1 Syntax

<pre>Prefix operator \::= \langle name \rangle \| (\langle expression \rangle) \| rhs \| \langle prefixed expression \rangle</pre>

<pre>Prefix expression \::= \langle prefixed operator \rangle[\langle expression \rangle_1,] \langle routine command \rangle \::= \langle name \rangle \| \langle prefixed expression \rangle</pre>

10.1.2 Semantics

A function is a representation of a rule for evaluating function calls, which are forms of expression (see Section 5.3). A routine is a representation of a rule for obeying routine calls, which are forms of command (see Section 9.4). Functions and routines fall into the following categories:

(a) Programmers' functions and routines: functions and routines introduced by an activation of one of the special forms for definition to be described in Section 10.3.

(b) Basic functions and routines: any of the functions and routines described in Appendix 3 to this manual, introduced into the program by use of the corresponding name, in a context where it is not subject to any definition of that name.

(c) Library functions and routines: an implementation dependent function or routine, introduced by an undefined occurrence of a name which does not correspond to any basic function or routine name.

It is assumed that any published program will be accompanied by an adequate, possibly informal account of these functions and routines. In particular, it must be possible to ascertain the data type of any expression involving a reference to such a function or routine.

All functions and routines are regarded directly as data items, of one of the function types (see Section 10.2.2) or of type routine.

A function call is any prefixed-expression in which the prefixed-operator is an expression of one of the function types. A routine call is a routine command consisting either of a single name of type routine, or of a prefixed-expression in which the prefixed-operator is an
expression of type routine. The single name routine call is interpreted as a call without parameters to the routine which is the value of the name. Other function and routine calls indicate an 'actual parameter list', which is the possibly empty list of expressions which follows the prefixed-operator, enclosed in square brackets.

To evaluate a function call, or to obey a routine call, the R-value of the function or routine is obtained, by evaluating the prefixed-operator of the prefixed-expression (or the single identifier, in the parameterless routine call). The rule represented by this R-value is then invoked using the ordered sequence of data items obtained by evaluating the actual parameter expressions for the call (in L-mode or in R-mode, depending on the mode of call of each parameter, see Section 10.3.3). In the case of a routine, this rule determines a sequence of commands to be obeyed. In the case of a function, the rule determines an evaluation process, from which a list of results is obtained. There are two forms of this rule; one determines how the evaluation is to be carried out in L-mode, and the other in R-mode. The choice between these forms is determined by the mode of evaluation of the function call as an expression.
10.2 Functions and Routines as Data Items

10.2.1 Syntax

(function type) ::= <single type>, function

<single type> ::= <data type>|<array type>|<function type>|(<single type list>)

<single type list> ::= <single type>,<single type>

10.2.2 Types

There is just one type of routine, the type routine.

There are an infinite number of types of function, two functions having the same data type if and only if they produce the same type of result, or produce results which match in number and in types, if they produce more than one result each.

As data items, those basic and library functions which are polymorphic as to their results (whose result types depend on the types of their parameters), are considered as functions of result type general, or (general, general), or (general, general, general) etc. depending on the number of results. Polymorphic functions with varying numbers of results are not considered as data items, and may only occur in the position of single name prefixed-operators.

Programmers' functions and routines may not be polymorphic. The numbers and types of results and parameters are specified in the special forms for function and routine definition (see Section 10.3).

The data type of a function is symbolised by preceding the basic symbol function by the result types in order, surrounded by parentheses and separated by commas if there is more than one result. Functions specified as type function, without any specification of result types, are assumed to have one result each, of the preferred type. Any of the data types in the result type list may itself be a functional type. For example, the following is an acceptable declaration:

let f be (real, index function, function) function

Values of f within the scope of this definition must be functions producing three results each; in order, a real, and index-valued function, and a function with a single value of the preferred type.
Transfer functions may be defined between those function types for which there are transfer functions between result and parameter types. None of these is implicitly invoked in cases of mismatching, e.g., if a function is assigned to a function variable of a different result type.

Note that the type of a function or routine does NOT depend on the number, types, or modes of calling of parameters, nor on whether the function or routine is fixed or free (see Section 10.3.4).

10.2.3 Expressions and Assignments

The R-value of an expression of a function or routine type is a function or routine which may be applied, assigned, or used as parameter in a function or routine call. At any point in a computation, any function or routine R-value which may arise must have originated in one of the following ways:

(a) as the R-value of a basic or library function or routine,

(b) as an R-value obtained from a call to a basic or library function whose result type involves a function or routine type,

(c) as the initial R-value of a function or routine identified defined previously using one of the special forms of function routine definition to be described in Section 10.3.

Such an R-value may be assigned to any function or routine variable for future assignment or application.

The rules for applying R-values of types (a) and (b) are to be obtained from the specifications of the basic or library functions or routines involved. The rules for applying R-values of type (c) are described in Section 10.3.

In a function or routine assignment, what is assigned is a representation of a computational rule; the rule itself remains unaltered. Thus, if \( f, g \) are function identifiers of the same result type, any sequence

\[ f := g ; b := g[a] \]

in a program can be replaced without altering the interpretation of the program, by the sequence

\[ f := g ; b := f[a] \]
10.2.4 Equality between Functions, Routines (see Note 1)

The only infixed relational operators between function expressions or between routine expressions are the operators

\[ = \neq \]

Before defining a notion of equality between function or routine R-values, it will be necessary to define a notion of EQUIVALENCE between two L-values of the same data type. This notion, and the notion of equality between function or routine R-values, will be seen to be interdependent.

Firstly, it should be noted that the only ways in which L-values may arise are the following:

(a) as constant L-values, i.e., as the L-values of constant unalterable R-values.
(b) as the L-values allotted to local variables, or to parameters called by value, or to a data item without an L-value (see Section 4.2).
(c) as results of calls to LH functions.
(d) as results of calls to basic or library functions.

There are four corresponding rules to define the equivalence of two L-values. Two L-values are equivalent if and only if they are of the same data type and satisfy one of the following conditions:

(a') they are both constant L-values, whose R-values are equal.
(b') they are both the same L-value, i.e., the L-value allotted at the same activation to a local variable or parameter called by value, or in an R-value to L-value transfer.
(c') they both result from calls to LH functions, and are compounded from load functions and update routines which are equal in the sense of this section.
(d') one L-value results from a call to a basic or library function, whose specifications, together with the rules of this section, imply that it is identical with the other L-value.

To define equality between two function or routine R-values, it is necessary, in the same way, to refer to the origins of the two R-values, as analyzed in Section 10.2.3. Two function or routine R-values are equal if and only if one of the following conditions hold:

(a') they are both R-values of the same basic or library function or routine.
(b') both R-values originate from the same special function or routine definition, possibly at different activations, and the L-values of corresponding free variables are equivalent. (see Note 2)
(c') one of the R-values results from a call to a basic or library function, whose specifications, together with the rules of this section, imply equality with the other R-value.

NOTES
1. This specification is given in its present form with an eye to compatibility with any extension of CPL which might deal with more complicated forms of data structure than are at present incorporated. These may possibly be viewed as types of free function, in which case the definition of equality between such functions is of importance. Until that time, the feature is of small significance, and deviant interpretations of this equality notion might not be disastrous. In particular (b') might be amended to:
   (b'') both R-values originate from the same activation of the same special function or routine definition.

   without the roof caving in.

2. In particular, if the two R-values are fixed, they are equal if and only if the R-values of all free variables were equal at definition time.
10.3 Function and Routine Definitions

10.3.1 Syntax

<lh function body> ::= <fix <definition>>,<
    load <block> update <block>

<function definition> ::= <recursive >,<variable | constant >,<
    name><[<formal parameter list>],<
    = | = >><expression list>><
    recursive >,<variable | constant >,<
    <fixed | free >,<function><name>
    [<formal parameter list>],<
    be
    <lh function body>><
    $ <lh function body> $>

<routine definition> ::= <recursive >,<variable | constant >,<
    <fixed | free >,<routine><name>
    [<formal parameter list>],<
    be <block>

10.3.2 Semantics, General

Functions and routines may be defined by type, or by value or by reference, in any of the standard ways described in Section 7. These forms of definition, however, can only create a new function or routine R-value by a call for one of the basic or library functions; in this case the computational rule specified by the R-value will be determined by the specification of the basic or library function which produces it. Definitions of the special forms of function and routine definitions whose syntax is given in Section 10.3.1 can also produce new function or routine R-values. Each instance of such a definition characterizes a computational rule, and, when activated, sets up a function or routine R-value which represents that rule.

Function and routine definitions have the following features:

(a) an identifier, the name which is the subject of the definition.
(b) a list of identifiers and basic symbols, following (a), enclosed in square brackets, and possibly empty; this is the FORMAL PARAMETER LIST for the function or routine definition.
(c) a **BODY**; that part of the definition following an occurrence of one of the basic symbols

```
be = =
```

The remaining parts of function and routine definitions have the following significances:

- **variable** signifies that the function or routine is a variable data item which may subsequently be updated. Otherwise the data item is assumed to be constant, with an unalterable R-value. The latter is also signified by using the basic symbol **constant** in the definition.

- **function**...**be** are used in LH function definitions (see Section 10.3.5).

- **routine**...**be** are used in routine definitions.

- **fixed** signifies that the LH function or routine is fixed (see Section 10.3.4).

- **free** signifies that the LH function or routine is free (see Section 10.3.4).

If neither **fixed** nor **free** occur in a LH function or routine definition, it is assumed to be free if it has any free variables, and fixed if it has none.

* = signifies an ordinary definition of a fixed function.

* = signifies an ordinary definition of a free function.

For ordinary function definitions, the body of the definition is an expression list; for routine definitions, the body is a command (in the form of a block). In these cases the rule for application of the corresponding function or routine is that the body of its definition is, respectively, evaluated or obeyed, special provision being made for the evaluation of any identifiers which are not defined within the body (which have FREE occurrences in the body, not in the scope of any definition of the identifier concerned within the body). These identifiers are either **FORMAL PARAMETERS** i.e., those identifiers whose names appear in the formal parameter list of the definition, or **FREE VARIABLES** i.e., any other identifiers having free occurrences within the body.
10.3.3 Formal Parameters

The identifiers in the formal parameter list are alloted a data type and mode of calling, by use of the basic symbols value and reference, and the data type specifiers described in Section 7.1. The type and mode of calling of any identifier are given by the nearest preceding type specifier and mode specifier (value or reference) in this list. Value signifies that the parameter is CALLED BY VALUE; reference signifies that it is CALLED BY REFERENCE. If there is no preceding type specifier in the list, the type of a parameter is taken to be the preferred type. If there is no preceding mode specifier, the mode is by value.

Before applying a function or routine definition, the formal parameters are provided with L-values, which bear the following relation to the corresponding actual parameters. If the parameter is called by value, its L-value is taken as a fresh L-value, with an initial R-value given by the R-value of the corresponding actual parameter expression. If the parameter is called by reference, it is given the L-value of the corresponding actual parameter expression in the call. (Note the similarity between parameters called by value and local variables defined using =, and between parameters called by reference and local variables defined by $\wedge$.)

Unless the prefix operator in the function or routine call is a name standing by itself, and occurs as the subject of a special function or routine definition with the attribute constant, the type of each actual parameter expression must be the same as that specified for the corresponding formal parameter in the formal parameter list. In the exceptional case, transfer functions are implicitly invoked where necessary (see Section 5.3.2).

Function and routine calls must result in the same number of actual parameter values as there are formal parameters in the formal parameter list.

10.3.4 Free Variables

The treatment of free variables depends on whether the function or routine is fixed or free. If it is free, the L-values of the free variables are taken to be precisely the L-values of these identifiers at the moment of activation of the definition. If the function or routine is fixed, the free variables are provided, at definition time, with L-values, whose R-values are constant, and are the R-values of those identifiers at definition time. If any free variable in the body of a fixed function or routine is a free array, the constant R-value is taken to be that of a fixed copy of the array, the elements of which are
unalterable (see Section 11.1.2). Fixed functions and routines may not have free variables which are either free functions or free routines or labels. Within the bodies of fixed functions and routines, no assignments may be made to free variables, or to elements of free variable arrays.

10.3.5 LH Functions

In the normal course of events, L-values are disjoint; that is to say, any assignment to one L-value does not affect the R-value associated with any other L-value. This is because L-values are ordinarily created either by activating a definition by type, or definition by =, or at a function or routine call, as the L-value of a parameter called by value, in which case the new L-value is always chosen to be disjoint from all previously created L-values.

In general, however, it is not adequate that any two L-values are either identical, or disjoint; it is desirable to have ways of creating new L-values, which may SHARE with previously created L-values in arbitrarily complex ways. These are provided by calls to LH functions, and by the special forms for definition provided by LH function definitions.

An L-value may be compounded from two data items of the following sorts:

(a) a parameterless function, of type $T_1$, function, for some data type specifier $T_1$; this produces an associated R-value, of data type $T_1$, when evaluated in R-mode; this is called the LOAD FUNCTION.

(b) a routine with one parameter called by value, of type $T_1$, this is known as the UPDATE ROUTINE, and, when called, it has the effect of changing the associated R-value to be the R-value of its actual parameter expression.

The result of a call to an LH function is an L-value compounded in this way from a function, defined by the block following the basic symbol load, and a routine defined by the block following the basic symbol update.

To be more precise, where the LH function definition reads:

```
...load C1, update C2 ... 
```

$C_1, C_2$, standing for blocks, the L-value produced by a call to that LH function is compounded from the function $f$ which would be defined by:
let \( f \) = \textbf{value of} \( C_1 \)

and the routine \( R \), which would be defined by:

\[ \text{let routine } R[T; x] \text{ be } C_3 \]

in place of that part of the body of the LH function definition. \( C_3 \) here is obtained by substituting \( x \) for the basic symbol \( \text{rhs} \) at all its occurrences in \( C_2 \) (outside any nested LH function definition), and \( T \), is the result type of \( f \).

The definition following the basic symbol \( \text{fix} \), if it occurs, is activated when the LH function call is evaluated, and has as its scope the remainder of the body. Its purpose is to make it possible to ensure that the \( L \)-value concerned (but not, of course, its associated \( R \)-value) is not altered by the side effects of other assignment statements between its evaluation at the application of the LH function and its possibly subsequent use to find the \( R \)-value or to update the \( L \)-value.

LH functions may only be single-valued; i.e., the result type of \( f \) as defined above must be a single data type, and not a data type list.

The result of evaluating a call to a LH function in \( R \)-mode is obtained by evaluating the call in \( L \)-mode, and then calling the load function to obtain the current associated \( R \)-value.

10.3.6 Determination of Result Types

The data types of functions introduced by special function definitions is determined from the definitions in the following ways:

(a) Non-recursive ordinary functions

The result type list for a non-recursive ordinary function definition is the ordered list of types of constituents of the expression list on the right hand side of the \( = \) or \( \equiv \) sign. The same rules apply for the types of numerical constants as in simple definitions using \( = \) (see Section 7.4.3).

(b) Non-recursive LH functions

Each result type list consists of a single type. When the body of the LH function definition reads:

\[ \ldots \text{load } C_1, \text{update } C_2 \ldots \]

with \( C_1, C_2 \) blocks, the type of the LH function is identical with that of the function defined by:
let f[] = value of C,

The type of rhs in C\alpha is taken as the result type of the function as determined in the above manner.

(c) Recursive functions

The result type list for a recursive function, or set of mutually recursive functions cannot necessarily be determined directly from (a) or (b) above, since the type of a defining expression may depend on one of the types to be determined. In this case, the result types are defined as follows:

Given any recursive set of definitions, an assignment of data types to the definitions is CONSISTENT if, assuming those data types on the right-hand side of definitions in the set, the data types for the definitions obtained by following (a), (b) above and the rule of Section 4 are transferable without loss of information to the assumed types. That is to say, corresponding data types that are not functional are directly transferable according to the rules of Section 3.3, and result type lists for functional data types have corresponding components that are also transferable without loss of information in this sense. If, for each definition of the set, there exists a data type which is transferable without loss of information to each data type corresponding to that definition in any consistent assignment of data types to the set, then that is taken as the data type for that definition, provided that in this manner a consistent assignment of data type is obtained. If no unique consistent assignment can be obtained in this way, the data types are implementation dependent.
11 ARRAYS

11.1 Subscripted Expressions. Arrays as Data Items
   11.1.1 Syntax
   11.1.2 Semantics

11.2 Basic Functions for Arrays
   11.2.1 Array-creating Functions
   11.2.2 Other Functions
11.1 **Subscripted Expressions, Arrays as Data Items**

11.1.1 **Syntax**

<prefixed operator> ::= <name> | (<expression>). | <prefixed expression>

<prefixed expression> ::= <prefixed operator>[<expression>]

<array type> ::= <single type>_<number> array | <single type> vector | <single type> matrix

11.1.2 **Semantics**

An array is a particular sort of function, of one *Index* parameter (by value) defined on just one closed segment of possible parameters. The free variables of an array are in one-to-one correspondence with the set of possible parameter values, and have mutually disjoint L-values; the result of evaluating a call to an array is that free variable which corresponds to the value of the parameter.

Arrays may be either free or fixed.

An R-value of one of the array-types is a representation of an array. The different possible array-types correspond to the different result types of the array, considered as a function, in the following manner:

If $T_i$ is a single-type, an R-value of type $T_i$ array or $T_i$ vector represents an array of result type $T_i$. If $T_i$ is omitted, the result type is taken as the preferred type.

The remaining result types may be deduced from the following synonyms:

- `matrix`, 2 array are synonymous with 1 array 1 array
- 3 array is synonymous with 1 array 1 array

and so on. For example, a `real 4 array` R-value represents an array with result type `real 1 array 1 array`, i.e., with result type `real 3 array`.

A subscripted expression is any prefixed-expression in which the prefixed-operator expression is of an array-type. If $A_i$ is an expression of some array-type, and $E_1, \ldots, E_n$ are expressions, then

$$A_i[E_1, E_2, \ldots, E_n]$$

is synonymous with
\[ A_i[E_1][E_2, \ldots, E_n] \]

The expressions separated by commas are the subscripts of the subscripted expression. A subscripted expression with just one subscript is evaluated in the same way as a function call (see Section 10). If there is more than one subscript, the expression is evaluated in accordance with the reduction to the one subscript case given above.

If the subscript value is outside the range of permitted values, the result is undefined.

The semantics of array expressions, assignments, etc., may be obtained by reference to the corresponding properties for functions (see Section 10.2.3, 10.2.4).

In particular, note the following points:

(a) No automatic copying of arrays is done on assignment. For example, after obeying
\[ A_i := B_i; B_i[i] := B_i[i] + 1 \]
\[ A_i[i] \] will have been altered.

(b) Note particularly that equality between arrays is defined in a very strong sense, which, for numerical vectors and matrices, does not coincide with ordinary mathematical usage. Two free arrays are equal only if their free variables have the same L-values; that is to say, only if both have been obtained by sequences of assignments of the same array (only if both are the same array, in every sense). For equality in the mathematical sense between free arrays, the Boolean function Equal must be used (see Appendix 3). Note that two fixed arrays are equal if they are equal in the weak mathematical sense.

There are no special forms available for the definition of arrays, nor are there transfer functions from function types to array types. The only ways in which arrays may be created are as the results of calls to certain basic functions (see Section 11.2.1).

N.B. (a) and (b) above will be seen to be intuitively more acceptable if an array is conceived of, not as the configuration of its elements, but as a POINTER to this configuration. Assignment and equality apply to these pointers rather than to the totalities of elements.
11.2 **Basic Functions for Arrays.**

11.2.1 **Array-Creating Functions**

(a) **Newarray**

A call has the form

\[
\text{Newarray}\left[T_i, E_i\right]
\]

where \(E_i\) is an expression list, and \(T_i\) is a data-type specifier. The function is VARIADIC; that is, it may be called with varying numbers of parameters. On evaluation \(E_i\) produces 2n R-values, \(A_1, B_1, \ldots, A_n, B_n\) of type \text{Index}. The value of the call is then a new array of type

\[T_i \text{ n array}\]

the L-values in which are disjoint from existing L-values. The values \(A_1, B_1, \ldots, A_n, B_n\) determine subscript bounds; after the assignment

\[N_i := \text{Newarray}\left[T_i, E_i\right]\]

the expression

\[N_i\left[E_{i1}, E_{i2}, \ldots, E_{in}\right]\]

is a subscripted expression all of whose subscripts are within bounds if and only if

\[A_i \leq X_i \leq B_i \quad \text{for } 1 \leq i \leq n\]

where each \(X_i\) is the R-value of \(E_{ii}\).

(N.B. Arrays need not be rectangular, although arrays directly produced by Newarray are.)

(b) **Formarray:**

A call has the form

\[
\text{Formarray}\left[T_i, E_i\right]
\]

with \(T_i, E_i\) as in (a). With the notation of (a), the value of the call is a function of type

\[T_1 \text{ n array function}\]

taking \(\prod_{i=1}^{n} (B_i - A_i + 1)\) arguments of type \(T_i\), which, at each call forms a new array; the R-values of actual parameters for the call will be the R-values of elements of the
resulting array arranged in lexicographic order with respect to subscripts. For example, after obeying

\[
N_i := \text{Formarray} \left[ \text{real}, (1,2), (1,2), (1,2) \right] \left[ ((E_{11}, E_{12}), (E_{12}, E_{12})), ((E_{21}, E_{22}), (E_{22}, E_{22})) \right]
\]

each \( N_i[l,j,k] \) has the R-value of \( E_{ij}^k \), for \( i, j, k = 1 \) or \( 2 \).

(Note the use of cosmetic parenthesis in the above example; these could be omitted without altering the effect of the command.)

(c) Copy

The value of \( \text{Copy}[E_i] \), with \( E_i \) an expression of some array-type, is a new free array of the same type, the same spectrum of legitimate subscript sequences, and the same R-values for corresponding elements as the value of \( E_i \). Elements which are arrays are copied.

(d) Fix

The value of \( \text{Fix}[E_i] \) is a fixed array of the same type as \( E_i \), the same spectrum of subscripts, and the same R-values for corresponding elements. \( E_i \) may be an expression of any array type, except for \text{label} \ n \text{ array}, (any \( n \)). The effect of \( \text{Fix} \) applied to an array containing a free function or routine is undefined.

11.2.2 Other Functions

(a) Bounds

A function of type \((\text{Index, Index}) \text{ function}\); taking one parameter of any array-type. The two values are, in order, the lower and upper bounds for single subscripts of the array (considered as a vector of subarrays).

(b) Equal

A function of type \text{boolean function}; taking two parameters of array-types. The value is \text{true} if and only if the two arrays are of the same type, have the same spectra of subscript sequences, and equal R-values in corresponding elements.

(N.B. \( \text{Equal} \left[ N_i, \text{Copy} \left[ N_j \right] \right] \) always has the value \text{true}, but \( N_i = \text{Copy} \left[ N_j \right] \) always has the value \text{false}, unless \( N_i \) is a null array.)
Appendix 1 The Preprocessor

Part 1 General Description

This is intended to be both a description of the transformation from an implementation alphabet to canonical CPL and a general example of a CPL program. The transformation is written in CPL and appears in Part 2; Part 3 then describes the functions and routines which are used but not declared in Part 2.

The routine Preprocessor uses recursion to model the bracketing structure of the source text; each level of recursion corresponds to a level of brackets. Its actual parameters are the Bracket type (e.g. $ ( \rightarrow )$, the section bracket tag if the bracket type is $ and a label to return to if a closing section bracket is found with a tag which does not match with the current section. It is thus easy to write the preprocessor rule concerned with the insertion of closing section brackets.

The removal of comment is done by the input routine NextChar and since comment is introduced by the pair of symbols || this routine requires a single character buffer which is called InputBuffer. The end of the source stream is recognised by a special end of stream character.

Most of the preprocessor rules are performed by the output routine Output. This routine makes use of the boolean variables TerminatorPending and TerminatorSuppressed to control the insertion or deletion of the command separator ';'. TerminatorSuppressed is true if the most recent symbol processed other than ';' and 'Newline' was the symbol c. TerminatorPending is true if a Newline or ';' has been read since the last canonical CPL symbol was output. The rule for insertion of terminators can most clearly be expressed as follows: a terminator is inserted between two canonical CPL symbols if a terminator is pending and not suppressed by c and if the first symbol can end a command and the second can start a command. It should be noted that all symbols which may end a declaration may also end a command. The last canonical CPL symbol output is always held in the string variable LastOutput. The rules for the insertion of do and the recognition of pos and neg are also incorporated in Output.

The working variable c, is used by Preprocessor and always contains the latest character produced by NextChar. Preprocessor is always called with c set to the next character. The routines ReadTag ReadUnderlinedWord and ReadSpaces all leave c correctly set.
Part 2    The Preprocessor Program

let c, s, s' all be string

let LastOutput, InputBuffer both = "" || i.e. the empty string

let TerminatorPending, TerminatorSuppressed = false, true

let rt NextChar [string ref c] be

§N.ch c := InputBuffer

Readchar [ CPL.Source, InputBuffer ]

If c = '|' = InputBuffer do
§ until c = '*'n do Readchar [ CPL.Source, c ]
Readchar [ CPL.Source, InputBuffer ] §N.ch

let rt Output [string x] be

§O.p if x = 'g' do § TerminatorSuppressed := true

return §

if TerminatorPending ∧ ¬ TerminatorSuppressed ∧
    CanEndCommand [LastOutput] ∧ CanStartCommand [x] do
 § Write [ ';' 'g' ]
 LastOutput := ';' §
 TerminatorPending, TerminatorSuppressed both := false
 x := MonadicTransformation [LastOutput, x]

if MustStartCommand [x] ∧ CannotProceedCommand [LastOutput]
do § Write [ 'do g' ]
 LastOutput := 'do' §

Write [ x <= 'g' ]
LastOutput := x §O.p

let rt ReadTag [string ref s] be

§R.T s := ""

§ NextChar [c]
unless Letter.Digit.Dot [c] break
 s := s <= c § repeat

until c ≠ '*'' do § s := s <= c
 NextChar [c] §R.T.
let rt ReadUnderlinedWord [string ref s] be
  s := c
  while UnderlinedLetter [c] do

let rt ReadSpaces [string ref c] be
  NextChar [c]
  if c ≠ 's' return repeat R.S

let rec rt Preprocessor [string BracketType, BracketTag, label MismatchReturn] be
  Switch : Jump on c into See Appendix 1 p. 8 for
case '$: ReadTag [s]
  Output ['$']
  Proprocessor ['$', s, Mismatch]
  goto Switch

  case ':': ReadTag [s]
    if BracketType ≠ '$' do Report [1]
  Mismatch : Output ['$']
    if BracketTag ≠ s goto MismatchReturn
    return

  case 'n':
    case '*':
      if TerminatorPending := true
      NextChar [c]
      goto Switch

  case '(': case 'p':
    Output [c]; s := c
    NextChar [c]
    Preprocessor [s, '(', Error]
    Error : goto Switch

  case ')':
    unless BracketType = '(' do Report [2]
    Output [c]; NextChar [c]
    return

  case ']':
    unless BracketType = '[' do Report [3]
    Output [c]; NextChar [c]
    return

  case ',':
    if BracketType = '->' do
      Output ['comma'] NextChar [c]
      goto Switch
      ReadSpaces [c]
      unless c = '.' do Output [','] NextChar [c]
      goto Switch
      Return
NextChar[c]
if Digit[c] do
  Output[',']
  while Digit[c] do
    s := s<>c
  NextChar[c]
  Output[','] <> s
  goto Switch
if s = 's' do ReadSpaces[c]
if c <> '.' do Report[4]
while c = '.' do ReadSpaces[c]
unless c = ',' do Report[5]
Output['dot string'] ; NextChar[c]
goto Switch

**case** 's' : s := 's'
NextChar[c]
until c = '?' do
  test EscapeCharacter[c]
  theno
    s := s<>SpecialChar[c]
  or do s := s<>c
  NextChar[c]
Output['s'] <> s) ; NextChar[c]
goto Switch

**case** '<' : NextChar[c]
if c = '<' do Output['<'] ; NextChar[c]
if c = '=' do
  NextChar[c]
    if c = '>' do Output['< = >'] ; NextChar[c]
    goto Switch
  Report[6]
Output['<']
goto Switch

**case** '>' : NextChar[c]
if c = '>' do Output['>'] ; NextChar[c]
Output['>']
goto Switch

**case** ':' : NextChar[c]
if c = '=' do Output['='] ; NextChar[c]
Output['=']
goto Switch
case 's' : case '_' : NextChar [c]
goto Switch

the rest : if BigLetter [c] do
  s := c
  ReadTag [s']
  Output ['x' => s => s']
goto Switch

if SmallLetter [c] do
  s := c
  NextChar [c]
  while c = '' do
    s := s => c
    NextChar [c]
  Output ['x' => s]
goto Switch

if Digit.Dot [c] do
  s := '!
  while Digit.Dot [c] do
    s := s => s
  Output ['x' => s]
goto Switch

if UnderlinedLetter [c] do
  s := Standard [s]
  while Combinable [s] do
    if c = 's' do ReadSpaces [c]
    unless UnderlinedLetter [c] do
      Output [s]
      goto Switch
    ReadUnderlinedWord [s']
    s' := Standard [s']
  test Standardizable [s => s']
  thendo s := standard [s => s']
  ordo Output [s]
  s := s'
Output [s]
goto Switch

if c = Endof StreamChar do
  unless BracketType = 'StreamStart' do
    return
Output [c]
NextChar [c]
goto Switch
Main program

LastOutput, InputBuffer both := ' '
NextChar [c] ; NextChar [c]
Preprocessor ['StreamStart', ',', Error] ; Error := finish

Part 3 Additional definitions.

A number of identifiers in Part 2 were left undefined; these are defined informally in this section.

a) The boolean function CanEndCommand [string LastOutput]
   is true if LastOutput is any of the following:
   Number, Name, StringConstant § ] ) :
   repeat break return rhs finish
   index integer / real etc.

b) The boolean function CanStartCommand [string x]
   is true if x is any of the following
   Number, Name, StringConstant
   pos, neg, + - ( §
   if unless while until test for forex
   break finish return resultis valof refof goto

c) CannotProceedCommand [string LastOutput] is true if
   LastOutput is any of the following:
   ; § do : or

d) MustStartCommand [string x] is true if x is any of the
   following:
   if unless while until test
   for forex return break finish
   resultis goto

e) The string function MonadicTransformation [string LastOutput, x]
   is used to recognise when + and - are monadic; if x is not + or - then
   the result is x, if LastOutput is not any of the following:
   number, name, ] , § , )
   and x is + or - then the result is pos or neg respectively.

f) Standard [string x] is a string function whose result
   is allbe if x is are or bothbe
   be const
do doforext
forext go
goto
or
rec
ref
refof
resultis
resultbe
rt
  to
  val
  valof

otherwise the result is x

g) Combinable [string x] is true if x is
   all d do double for
   go long or ref
   result repeat val

h) Standardizable [string x] is true if
the result of Standard [x] is an underlined word in the basic
symbol set of canonical CPL.

i) Letter.Digit.Dot [string c] is true if c is a
   letter or a digit or a dot.

j) Digit.Dot [string c] is true if c is either a
digit or a dot.

k) Digit [string c] is true if c is a digit.

l) EscapeCharacter [string c] is true if c is
   the escape character for strings.

m) SpecialChar [string c] transforms the special
character which occurs after the escape character in strings
to the string character it represents.

   e.g. the result is the character 'Newline' if c is n
       'Backspace'
       'Erase'
       etc.
The switch-command

Syntax

\[
\text{<switch command> ::= switch on <expression> into <block>}
\]

\[
\text{<label> ::= <identifier> : | case <constant> | default :}
\]

The synonyms for switch on are:
- caseswitch and jump on

and for default are:
- the rest and others

Semantics

The switch-command is a convenient way of controlling a multiway decision in CPL; it is designed as an extension to the test-command which switches to one of two alternatives depending on the value of a boolean expression.

The type of the switch is the type of its controlling expression and this may be string or any of the numerical or logical types. A case-label is said to belong to the smallest enclosing lexicographic switch-command and its constant is transformed to the type of this switch-command.

To execute a switch-command first the controlling expression is evaluated in R mode and then execution is resumed at the case-label which belongs to the switch-command and whose constant equals this value, or if no matching case exists execution continues at the command labelled default: . The constants of two case-labels belonging to the same switch-command may not be equal.

The switch-command has not yet been incorporated in The rest of the reference manual. It is intended to include it in the language.
APPENDIX A

List of Changes Foreseen in CPL

A. Alterations

1. Scope of where-clauses

This is somewhat confusing and unsatisfactory at present. We shall probably distinguish between an "expression-where" and a "command-where" (which probably includes a "definition-where") by some syntactic device similar to that which picks out the conditional commas and replaces them at the preprocessor stage by comma.

2. result is, return and break

"Detached" uses of these will be allowed with a scope which is lexicographically determined. These correspond, inter alia, to Landin's "program points".

3. finish

This exists only in the Elementary Manual. Its use will be extended and generalised.

B. Additions

1. Character

We hope to introduce a type character into the language.

2. Switch-commands

The switch-command described on p.8 of Appendix 1 will be added.

3. Table look-up

Commands for Table look-up seem to be desirable. These may prove to be part of the operations on sets (see next section).

C. Extensions

1. Compound Data Structures

The entire area of Compound Data Structures and related concepts needs to be added to CPL. This will involve a considerable extension of the concept of type. An outline scheme is already under consideration, but much more work is needed before it can be regarded as good enough for incorporation in the language.